Does extreme internal-wave breaking matter for ocean mixing?

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Abstract. Over the last few decades complex yet promising statistical models of internal wave breaking have been developed and favorably compared with a variety of oceanic data. The models equate the turbulent dissipation rate with the rate of downscale energy transfer through a steady spectrum. However, a recent profusion of strong wave breaking examples have appeared that violate statistical assumptions of the dissipation model yet produce significant turbulent mixing. Here we review a few such examples and discuss implications for development of accurate yet practical mixing parameterizations.

Introduction

Internal gravity waves are ubiquitous in the ocean, with typical horizontal velocities of cm/s and vertical displacements of tens of meters. Internal waves can be "extreme" in a variety of ways. Consider the favored definition of extreme as "rare but influential" where influence is of course inherently subjective. For example, some internal waves are extremely large in amplitude - waves observed in Mamala Bay (less than 10 km from the AHA meeting site) have vertical displacements of 100 meters and pose a significant potential complication for sewage outfall containment [Alford et al., 2006]. Some internal waves are extremely powerful - large-amplitude internal solitons pose a substantial risk to offshore oil platforms [Cai et al., 2003]. However, internal waves are often perceived as most influential for the turbulent mixing their death throes produce. Here we will focus on this internal wave driven mixing in the global ocean, and rare but influential properties thereof.

In fact, diapycnal mixing in the ocean interior is primarily driven by internal wave breaking. This mixing transports heat, dissolved gasses, and nutrients, controls exchange between the deep and upper ocean. Recent thinking suggests that the downward turbulent diffusion of heat in low latitudes provides potential energy that drives the global-scale Meridional Overturning Circulation [*Munk and Wunsch*, 1998]. In regional and large-scale numerical models turbulent mixing is typically represented by a diffusivity that is constant everywhere except possibly right below the surface mixed layer. However, mixing in the real ocean is far from spatially uniform, but has been shown to be patchy in space and time, which in turn has profound implications for patterns of biological productivity and ocean circulation [*Simmons et al.*, 2004].

Spectral theories of internal wave breaking

Understanding, predictive knowledge and eventual practical parameterization of mixing driven by internal wave breaking requires mastery of three steps.

- 1. Patterns of internal wave generation: Most energy input into the internal-wave field is in two frequencies of waves, tidal (generated by the sloshing of the barotropic tide over topography), and near-inertial (generated by surface winds). Global maps of estimates of both are appearing in the literature [*Egbert and Ray*, 2001; *Alford*, 2003]. For internal tides, while higher mode waves dissipate near the generation site (leading to a global pattern of mixing hotspots that mirrors that of internal tide generation), 70-90 percent of the energy escapes to propagate up to thousands of km across ocean basins as low-mode waves [*St. Laurent and Nash*, 2004].
- 2. **Pathways of propagation**: Propagating waves are affected by a range of processes, from refraction by the the evolving mesoscale to reflection and scattering from topography [*Johnston et al.*, 2003]. However, in much of the ocean evolutionthe lowest (energy-containing) modes seems reasonably well explained by simple propagation theory [*Rainville and Pinkel*, 2006].
- 3. Processes that transfer energy into small-scale turbulent mixing: Turbulent mixing is directly produced by the breaking of small-scale internal waves through shear or convective instability [*Staquet and Sommeria*, 2002]. However, the rate limiting step is often the rate at which energy is transferred from larger-scale waves to smaller-scale waves (which subsequently break). Though down-scale energy transfer within the internal wave field can be accomplished by a variety of means,

the focus here is on wave-wave interaction, which has been argued to dominate scale transformation away from boundaries [*Müller et al.*, 1986].

The statistical models that are typically used to predict down-scale energy transfer were inspired by the observation that spectra of internal wave energy as a function of frequency and wavenumber show a remarkable degree of similarity around the world, especially given the abovementioned inhomogeneity in forcing [*Garrett and Munk*, 1979]. This is not to say that the energy density is the same everywhere, but the energy density spectrum appears to largely follow a well prescribed family of shapes, with frequency and vertical wavenumber spectral slopes are both around -2. The vertical wavenumber spectrum rolls off at scales where the Richardson number is order one, suggesting wave breaking plays a dynamical role in maintaining the spectral shape [*Gargett et al.*, 1981; *Polzin*, 2004]

Energy may be transferred between finite amplitude waves of different scales through the nonlinear advection terms in the equations of motion. Full analytical solutions of nonlinear interactions for a broad-banded spectrum of waves are beyond easy reach, and theory has approached the problem through several types of statistical approximation [Müller et al., 1986; Lvov and Tabak, 2001; Polzin, 2004]. The strongest thread of analysis has been resonant interaction theory, which considers slow changes in the amplitude of freely propagating waves through first order expansion of nonlinear terms. The basic building block is the resonant interaction that occurs among three waves with sum and difference phases that add together ($k_1 \pm k_2 = k_3, \ \omega_1 \pm \omega_2 = \omega_3$.) The total rate of down-scale energy transfer as taken as the integral over all possible resonant and slightly off-resonant triads. Simplifying assumptions require waves to be incoherent, independent, and describable by a smooth spectrum. The dominant energy transfer to small-scale (nearbreaking) waves comes from scale-separated interactions high mode (shear-containing) waves interacting with lowfrequency, low-mode (energy-containing) waves.

Another approach to down-scale energy transfer uses raytracing to focus on scale-separated interaction between large and small-scale waves [*Henyey et al.*, 1986]. With similar assumptions of statistical independence (although weakened restriction on weakness of nonlinearity), the method predicts the wavenumbers of high-mode waves in essence randomwalk their way to smaller scales.

In both cases a prediction of steady down-scale transport (equated to the dissipation rate) can be written as a function spectral energy level, with the two approaches producing functionally similar formulae. This in turn can be turned into a useful tool to estimate the turbulent dissipation rate from low-resolution shear and strain measurements, which are far easier and cheaper to make than microstructure estimates of mixing [*Gregg*, 1989; *Polzin et al.*, 1995; *Gregg* *et al.*, 2003]. The end product dissipation parameterization is often referred to as the Gregg-Henyey (GH) scaling.

The next step is to turn such a theory into a predictive mixing parameterization that can be practically used in larger-scale numerical models that are starting to explicitly resolve the low-mode portion of the internal wave field. The above description suggests the field is well on its way to creating just such a thing, and perhaps the problem has moved from a question of fundamental science to one of practical implementation.

Trouble in paradise

However, after several decades of intense work to put together a model of wave dissipation, over the last few years there seems to have been a perverse rise of exceptions to the steady statistical view. In some cases mixing is extremely large, in others the magnitude is moderate but the turbulence departs from the GH model in ways that are extremely significant or unusual. Here we provide a quick tour of such deviatory behavior. The examples below are meant to be illustrative but are far from comprehensive.

Directly breaking waves - no cascade needed

The statistical theory outlined above assumes that the rate at which energy is made available for mixing is dynamically controlled by the rate of energy transfer through a reasonably broad continuum of internal waves between the largest and smallest (order one Ri) scales. Yet a number of examples have recently emerged of cases where low-mode waves, recently generated by external forcing, have order one Richardson numbers and are directly breaking.

A popular example is the tidal soliton, commonly observed in coastal seas around the world [Apel et al., 1995]. Typical soliton packets are formed by nonlinear steepening of an onshore propagating internal tide and contain a series of very high frequency (tens of minutes or less) sharp thermocline displacements and associated high shear and strong mixing. Figure 1 shows a soliton packet observed on the New England Shelf - turbulent dissipation is elevated orders of magnitude above background values on the isopycnals that follow the peak shear of each soliton pulse [MacKinnon and Gregg, 2003]. Moum et al. [2003] observe regular large solitons propagating onshore on the Oregon shelf and present stunningly detailed pictures of Kelvin-Helmholtz billows within. More than just dramatic dynamical oddities, soliton turbulence contributes the majority of turbulent dissipation for both regions, over a range of thermocline isopycnals that are crucial for transport of biological nutrients into the euphotic zone. Interestingly, though solitons also occur in deeper water, the stretched out low-mode waves don't always have enough shear to create significant turbulence [Klymak et al., 2006].



Figure 1. Turbulent mixing during the passage of one soliton packet on the New England Shelf. The solid lines are contours of northward (on-shelf) baroclinic velocity from -0.3 to 0.3 m s^{-1} in intervals of 0.1 m s^{-1} . The shaded areas are 4-m shear variance, ranging from 0 (white) to $3.5 \times 10^{-3} \text{ s}^{-2}$ (black) in increments of $5 \times 10^{-4} \text{ s}^{-2}$. Profiles of dissipation rate are overlain, and correspond to the colorbar above. The slight slant of each profile represents the passage of time as the profiler descends. The black (upper) and magenta (lower) stars on each profile indicate the evolving locations of the 22.65 and 24-kg m⁻³ isopycnals, respectively. Measurements were taken in 70 m of water. Velocity measurements from shipboard ADCO, isopyncal displacements and turbulent dissipation rate from the Modular Microstructure Profiler, see *MacKinnon and Gregg* [2003] for more details.



Figure 2. Average vertical profiles of turbulence dissipation near Kaena Ridge, Hawaii, for four time periods [*Klymak et al.*, 2007]. In each panel the thick black line is the estimate from the microconductivity probe, thin shaded from density overturns, thick shaded from Gregg-Henyey parameterization.

Extreme internal wave breaking

Yet some deep-water internal waves are also prone to direct breaking. For example Klymak et al. [2007] carefully delineate two regimes of internal-wave related turbulence on the Hawaiian ridge (Fig. 2). In the upper water column, wave amplitudes are moderate and observed turbulence agrees well with the GH model described above. Yet in the lower water column, large amplitude waves triggered by the barotropic tide are directly breaking through convective instability, and observed dissipation rates are 10-100 times larger than would be predicted by the GH statistical theory (Fig. 2, thick black versus grey). Such strong mixing surely matters for diffusion along the Hawaiian Island Chain, but cannot be represented by existing statistical theory. Recent observations on the Oregon slope show similarly strong overturns related to barotropic tidal flow along corrugated topography, interestingly in a region not predicted to be be a significant generation site for internal tides [Nash et al., 2007].

Finally, areas with energetic low-mode waves or relatively shallow water depths may violate GH approach because the bandwidth between large-scale and breaking waves is not wide enough to support the weakly nonlinear dynamics that set the rate of downscale energy transport in spectral theories. For example, even when solitons are not present on the New England Shelf, the "background" dissipation rate does not scale with shear and stratification in a way predicted by GH theory. Similar observations were made on the the Monterey shelf [Carter et al., 2005] and the Oregon shelf [Avicola et al., 2007]. At more intermediate depths, Kunze et al. [2002] observe dissipation rates in Monterey Canyon 30 times those predicted by GH-type scalings. They attribute the discrepancy to a combination of lack of spectral bandwidth and other topographic scattering processes that may accelerate the transfer of energy to smaller scales. Work is progress by the present author is attempting to numerically flush out the role of limited bandwidth for down-scale energy transfers.

Spectral transfer short-circuited

The GH statistical model of wave dissipation assumes that waves are statistically independent and incoherent, an assumption often made for dispersive wave fields. In particular, it is assumes that waves involved lose coherency on a time-scale shorter than the characteristic duration of significant energy transfer [*Müller et al.*, 1986]. However, recent evidence indicates that rapid Parametric Subharmonic Instability (PSI) may violate these assumptions. The process involves a transfer of energy from a propagating internal tide to smaller scale waves of near half the frequency. Numerical simulations by *MacKinnon and Winters* [2005] show that for a coherent internal tide (as often observed in the ocean), energy transfer from PSI can be orders of magnitude faster than predicted by statistical theory [*Olbers and* 5



Figure 3. IWAP: Time series of north-south velocity from HDSS sonar at 28.9N. Velocity is presented in a semi-lagrangian coordinate system using isopycnal locations measured by a Fast-CTD system.[*Alford et al.*, 2007]

Pomphrey, 1981]. Similar results have been found in numerical simulations by *Hibiya et al.* [1998, 2002]. *MacKinnon and Winters* [2005] predict that the transfer should be particularly efficient at a 'critical' latitude of 28.9, where the halffrequency waves are exactly the local inertial frequency. At this point the mechanism switches from a triad interaction between freely propagating waves to a pure instability, the time-scale of which is given in *Young and Tsang* [2007].

The Internal Waves Across the Pacific (IWAP) Experiment was designed to track long range propagation of the internal tide from generation at the Hawaiian Ridge to 37 N, and look for evidence of PSI along the way. The experiment involved a suite of measurements made from April-June 2006, including ship-based sonar measurements of velocity, 50-day time series with moored profilers, and short intensive time series with the Pinkel Fast-CTD [Alford et al., 2007]. Our initial conclusion is that while PSI does not significantly disrupt the propagation of the internal tide, it does provide a substantial source of near-inertial waves at the critical latitude. A 5-day time series at 28.9 shows clear nearinertial motions (Fig. 3). In striking contrast to more typical surface-generated (downward-propagating) near-inertial waves observed at other stations, these motions show no vertical phase propagation. This feature is consistent with generation through PSI, which produces similar amounts of up-going and down-going near-inertial energy. These inertial features have Richardson numbers around 0.7, suggesting wave stability restrictions may set the vertical wavelength selected by the PSI. Preliminary calculations suggest the power input into near-inertial waves through this mechanism is the same order or larger than wind input into the near-inertial wave field at this latitude.

This rapid energy transfer is not just of local interest, but appears to set large-scale latitudinal patterns of wave energy



Figure 4. IWAP Top panel: depth-averaged horizontal kinetic energy in clockwise (blue) and counter-clockwise (red) motions. Velocity was vertically detrended to remove the (low-mode) internal tide and emphasize shear-containing near-inertial motions. The solid line in each case indicates an average over 4-8 North-South transects with the shipboard HDSS sonar, the shaded areas indicated one standard deviation on either side. Lower panel: Turbulent diffusivity calculated using the method of *Kunze et al.* [2006]. Diffusivity was averaged only below 500 meters due to as yet unresolved instrument issues in shallower water.[*Alford et al.*, 2007]

and dissipation rate. The influence of PSI-generated waves may be seen by decomposing horizontal velocity into components that rotate clockwise with depth (indicating downward energy propagation), and counter-clockwise with depth (indicating upward energy propagation). Up-going energy rises sharply just south of the critical latitude (Fig. 4, upper panel). As a result of this elevated shear, turbulent diffusivity also rises in this latitude range (Fig. 4, lower panel). Further evidence of a strong PSI-induced latitudinal pattern of mixing comes from altimetric analysis [*Tian et al.*, 2006] and XCP surveys [*Hibiya and Nagasawa*, 2004].

Spectral myths

The most systematic challenge to spectral models of down-scale energy transfer is the suggestion that a universal steady frequency-wavenumber spectrum may not exist. Problems with and caveats to the GM spectrum have been noted from the beginning, such as coherency of the energycontaining motions, particularly the internal tide, and issues of 'fine-scale' contamination at higher frequencies and wavenumbers [*Müller et al.*, 1986]. More recently, *Pinkel* [2007] has proposed a kinematic model that goes further to explain a significant portion of the continuum frequency spectrum of shear as simply doppler shifting of a few intrinsic spectral peaks. In particular, he considers a world with intrinsic peaks near the inertial frequency and zero frequency (vortical mode), with a certain vertical wavenumber distribution. His kinematic model advects these motions by a combination of random lateral and vertical motions (as from, for example, mesoscale currents or instrument motion), and deterministic tidal heaving. The result is an apparent frequency spectrum that replicates several features of observations - an hourglass shape of increasing frequency bandwidth with increasing wavenumber, and realistic spectral levels and slopes of shear variance. The upshot is that we may be able to simplify the dimensions of the problem from a complicated spectrum of interacting waves (and associated weighty integrals over all interacting components) to a cast of only a few characters. If most of the down-scale energy transfer is happening at near-inertial frequencies, conceivably "assisted" by higher-frequency waves, a simplified model of wave dissipation could be developed.

Conclusions

At this point we must confront the following questions: **Does extreme internal wave breaking matter to mixing? Are statistical models of down-scale energy flux useful? Where do we go from here?** The answer in each case has two facets - a dynamical answer (what is nature actually doing) and a practical answer (how can we incorporate the most accurate mixing into large-scale numerical models). Consider the following two regimes.

Topographically complex or shallow areas

In shallow water and regions of complex topography (e.g. internal tide generation sites) it appears we don't yet have a full grasp on zoo of processes controlling turbulent mixing - new and strange beasts seem to surface at every AGU meeting. These include not only various nonlinear types of internal waves (solitons and bores), but also processes like topographic scattering that directly transfer energy to smaller scales [Müller and Liu, 2000; Kunze et al., 2002; Johnston et al., 2003; Nash et al., 2004] and hydraulic effects that are not really propagating waves at all [Thurnherr, 2006]. For many of these regions we seem to sill be in a phase of basic exploration, hopefully fueled by continuing observations. At the same time, the 'background' mixing in coastal areas does appear to show reliable statistical relationships with broad internal wave properties, although the limited bandwidth makes for a different sort of relationship than observed in the open ocean [MacKinnon and Gregg, 2003; Carter et al., 2005; Avicola et al., 2007].

Practically speaking, the best best for parameterizing mixing in such areas in the near term may be to use local high resolution models (e.g. *Fringer et al.* [2006]) that can resolve many of the small-scale non-hydrostatic processes. Existing and new observations provide an essential guide as to what processes should be included and focused upon. Such models can be run with an ensemble of initialized mesoscale states and analyzed to produce an ensemble-

averaged resultant diffusivity. This spatially variable but static diffusivity can then be plugged into operational regional models.

Deep open ocean

However, in large parts of the open ocean it appears that the wave spectrum (to the extent it is not a doppler shifted myth) is well-behaved enough that statistical ideas of wavewave interaction may indeed regulate the rate of energy transfer to dissipative scales. Strong breaking in places like the Hawaiian ridge is of local but probably not basin-wide significance. There are still some processes to be understood (e.g. broad latitudinal patterns set by PSI), but remaining dynamical issues may be able to be flushed out with a few focused observational programs and dedicated numerical process studies.

Practical parameterizations of open ocean mixing could take a couple of forms. One is a similar approach to above - taking an ensemble of mesoscale initial states and running an offline 'internal wave' model that generates internal waves, propagates the low modes, and uses an updated GHtype of parameterization to see where the dissipate [Müller and Natarov, 2003; Polzin, 2004]. The average diiffusivity map calculated from these processes could be fed back into a GCM . However, there is also the possibility of developing a more dynamic mixing parameterization that would prescribe diffusivity based on the low-mode internal waves that are already starting to be seen in higher-resolution GCMs (although most do not yet include tides). This approach would have the huge advantage that diffusivity could vary with seasonally changing stratification and mesoscale currents, and include the episodic nature of storm-induced nearinertial wave generation. Furthermore, such a dynamic mixing parameterization would allow for potentially important feedbacks involving things like storm intensity, stratification, mixing and overturning circulation strength in future climate predictions.

One stumbling block in this route is that GH-type parameterizations are usually not presented in a prescriptive form. That is to say they are designed to describe the rate of down-scale energy transport (dissipation rate) for a given observed spectrum. What is needed is a form that will take as a starting point the amplitude of a traveling low-mode internal tide or near-inertial wave and prescribe both the local spectrum that sets up in equilibrium with the power sucked out of the passing wave, and the rate of down-scale transfer to turbulence that such a spectrum mediates. Ideally some approximation of those dynamics could be encapsulated in a parameterization that is computationally simple and takes as input only variables like shear and stratification that GCMs already have on hand. Such an approach seems challenging but not unfeasible and efforts such as Müller and Natarov [2003], Polzin [2004], and work underway by the present Acknowledgments. This work brought to you by NSF (OCE 0425283) and ONR (N00014-05-1-0491). Matthew Alford and Rob Pinkel deserve much acclaim for the success of the IWAP experiment. Jody Klymak generously provided Figure 2 and associated dynamical insights. Finally, many thanks to the AHA conveners for the invitation and for organizing such an diverse and illuminating workshop.

References

- Alford, M. H., Energy available for ocean mixing redistributed though long-range propagation of internal waves, *Nature*, 423, 159–163, 2003.
- Alford, M. H., M. C. Gregg, and M. A. Merrifield, Structure, propagation, and mixing of energetic baroclinic tides in Mamala bay, Oahu, Hawaii, J. Phys. Oceanogr., 36, 997–1018, 2006.
- Alford, M. H., J. A. MacKinnon, Z. Zhao, R. Pinkel, J. Klymak, and T. Peacock, Internal waves across the Pacific, 2007, in prep.
- Apel, J., L. Ostrovsky, and Y. A. Stepanyants, Internal solitons in the ocean, *Tech. rep.*, Applied Physics Laboratory, Johns Hopkins University, 1995, report MERCJRA0695.
- Avicola, G. S., J. N. Moum, and J. Nash, A turbulence parameterization scheme for a coastal internal wave field in the presence of thermal wind shear, 2007, in prep.
- Cai, S., X. Long, and Z. Gan, A method to estimate the forces exerted by internal solitons on cylindrical piles, *Ocean Engineering*, 30, 673–689, 2003.
- Carter, G. S., M. Gregg, and R.-C. Lien, Internal waves, solitary waves, and mixing on Monterey Bay shelf, *Continental Shelf Research*, 25, 1499–1520, 2005.
- Egbert, G. D., and R. D. Ray, Estimates of M2 tidal energy dissipation from TOPEX/Poseidon altimeter data, J. Geophys. Res., 106, 22,475–22,502, 2001.
- Fringer, O. B., M. Gerritsen, and R. L. Street, An unstructured-grid, finite-volume, nonhydrostatic, parallel coastal ocean simulator, *Ocean Modelling*, 14, 139–278, 2006.
- Gargett, A. E., P. J. Hendricks, T. B. Sanford, T. R. Osborn, and A. J. Williams, III, A composite spectrum of vertical shear in the upper ocean, *J. Phys. Oceanogr.*, 11, 1258–1271, 1981.
- Garrett, C., and W. Munk, Internal waves in the ocean, *Ann. Rev. Fluid Mech.*, 11, 339–369, 1979.
- Gregg, M. C., Scaling turbulent dissipation in the thermocline, J. Geophys. Res., 94, 9686–9698, 1989.
- Gregg, M. C., T. B. Sanford, and D. P. Winkel, Reduced mixing from the breaking of internal waves in equatorial waters, *Nature*, 422, 513–515, 2003.
- Henyey, F. S., J. Wright, and S. M. Flatté, Energy and action flow through the internal wave field, *J. Geophys. Res.*, 91, 8487– 8495, 1986.
- Hibiya, T., and M. Nagasawa, Latitudinal dependence of diapycnal diffusivity in the thermocline estimated using a finescale parameterization, *Geophysical Research Letters*, *31*, 2004.
- Hibiya, T., Y. Niwa, and K. Fujiwara, Numerical experiments of nonlinear energy transfer within the oceanic internal wave spectrum, J. Geophys. Res., 103, 18,715–18,722, 1998.
- Hibiya, T., M. Nagasawa, and Y. Niwa, Nonlinear energy transfer within the oceanic internal wave spectrum at mid and high latitudes, J. Geophys. Res., 107, 2002.

- Johnston, T. S., M. A. Merrifield, and P. Holloway, Internal tide scattering at the Line Islands Ridge, J. Geophys. Res., 108, doi:10.1029/2003JC001.844, 2003.
- Klymak, J., R. Pinkel, C.-T. Liu, A. Liu, and L. David, Prototypical solitons in the South China Sea, *Geophys. Res. Lett.*, 33, 2006.
- Klymak, J. M., R. Pinkel, and L. Rainville, Direct breaking of the internal tide near topography: Kaena Ridge, Hawaii, J. Phys. Oceanogr., in press, 2007, submitted.
- Kunze, E., L. K. Rosenfeld, G. S. Carter, and M. C. Gregg, Internal waves in monterey submarine canyon, *J. Phys. Oceanogr.*, 32, 1890–1913, 2002.
- Kunze, E., E. Firing, J. M. Hummon, T. K. Chereskin, and A. M. Thurnherr, Global Abyssal Mixing Inferred from Lowered ADCP Shear and CTD Strain Profiles, *J. Phys. Oceanogr.*, 36, 1553–1576, 2006.
- Lvov, Y. V., and E. G. Tabak, Hamiltonian formalism and the Garrett-Munk spectrum of internal waves in the ocean, *Physi*cal Review Letters, 87, 2001.
- MacKinnon, J., and M. Gregg, Mixing on the late-summer New England shelf solibores, shear and stratification, *JPO*, *33*, 1476–1492, 2003.
- MacKinnon, J. A., and K. Winters, Subtropical catastrophe: significant loss of low-mode tidal energy at 28.9 degrees, *Geophysical Research Letters*, 32, doi:10.1029/2005GL023,376, 2005.
- Moum, J., D. Farmer, W. Smyth, L. Armi, and S. Vagle, Structure and generation of turbulence at interfaces strained by internal solitary waves propagating shoreward over the continental shelf, *J. Phys. Oceanogr.*, 33, 2093–2112, 2003.
- Müller, P., and X. Liu, Scattering of internal waves at finite topography in two dimensions. part I: Theory and case studies, *J. Phys. Oceanogr.*, 30, 532–549, 2000.
- Müller, P., and A. Natarov, The internal wave action model iwam, in *Near-Boundary Processes and Their Parameterization, Proceedings of the 13th 'Aha Huliko'a Hawaiian Winter Workshop*, edited by P. Müller and D. Henderson, 2003.
- Müller, P., G. Holloway, F. Henyey, and N. Pomphrey, Nonlinear interactions among internal gravity waves, *Rev. Geophys*, 24, 493–536, 1986.
- Munk, W., and C. Wunsch, Abyssal recipes II: Energetics of tidal and wind mixing, *Deep-Sea Res. Part I*, 45, 1977–2010, 1998.
- Nash, J., M. H. Alford, E. Kunze, K. Martini, and S. Kelly, Hotspots of deep ocean mixing on the oregon continental slope, *Geophys. Res. Lett.*, 34, doi:10.1029/2006GL028,170, 2007.
- Nash, J. D., E. Kunze, J. M. Toole, and R. W. Schmitt, Internal tide reflection and turbulent mixing on the continental slope, *J. Phys. Oceanogr.*, 34, 1117–1134, 2004.
- Olbers, D., and N. Pomphrey, Disqualifying two candidates for the energy balance of oceanic internal waves, J. Phys. Oceanogr., 11, 1423–1425, 1981.
- Pinkel, R., Advection, phase distortion, and the frequency spectrum of fine-scale fields in the sea, J. Phys. Oceanogr., in press, 2007.
- Polzin, K., Idealized solutions for the energy balance of the finescale internal wave field, J. Phys. Oceanogr., 34, 231–246, 2004.
- Polzin, K. L., J. M. Toole, and R. W. Schmitt, Finescale parameterizations of turbulent dissipation, *J. Phys. Oceanogr.*, 25, 306– 328, 1995.
- Rainville, L., and R. Pinkel, Propagation of low-mode internal waves through the ocean, J. Phys. Oceanogr., 36, 1220–1236, 2006, submitted.

- Simmons, H., S. Jayne, L. S. Laurent, and A. Weaver, Tidally driven mixing in a numerical model of the ocean general circulation, *Ocean Modelling*, 6, 245–263, 2004.
- St. Laurent, L., and J. Nash, An examination of the radiative and dissipative properties of the internal tides, *Deep-Sea Res.*, 51, 3029–3042, 2004.
- Staquet, C., and J. Sommeria, Internal gravity waves: from instabilities to turbulence, Ann. Rev. Fluid Mech., 34, 559–593, 2002.
- Thurnherr, A. M., Diapycnal mixing associated with an overflow in a deep diapycnal mixing associated with an overflow in a deep submarine canyon, *Deep-Sea Res II*, 53, 194–206, 2006.
- Tian, J., L. Zhou, and X. Zhang, Latitudinal distribution of mixing rate caused by the M2 internal tide, J. Phys. Oceanogr., 36, 35– 42, 2006.
- Young, W., and Y. Tsang, Near-inertial parametric subharmonic instability, 2007, submitted.
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