

Oceanic Internal Waves: global patterns and open questions

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Internal wave equations for the ocean

$$\frac{\partial \vec{u}}{\partial t} = -\vec{u} \cdot \nabla \vec{u} + 2\Omega_{\perp} \times \vec{u} - \frac{1}{\rho_0} \nabla p - g\hat{z} + \nu \nabla^2 \vec{u}$$

$$\frac{\partial \rho}{\partial t} = -w \frac{\partial \bar{\rho}}{\partial z} + \kappa \nabla^2 \rho$$

$$\nabla \cdot \vec{u} = 0$$

Boussinesq
 $\rho = \rho_0 + \overline{\rho(\mathbf{z})} + \rho'(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{t})$
 $\rho', \overline{\rho(\mathbf{z})} \ll \rho_0$

Traditional approximation

Incompressible

Inviscid (at least until they break)

Linear (surprisingly often, some nonlinear examples shortly)

Internal wave equations

Vertically propagating waves

Try a solution of the form

$$\mathbf{u}(\mathbf{x}, \mathbf{y}, \mathbf{z}, t) = \hat{\mathbf{u}} e^{-i[\mathbf{k}\mathbf{x} + \mathbf{l}\mathbf{y} + \mathbf{m}\mathbf{z} - \omega t]}$$

Get polarization and dispersion relationships

$$\omega^2 = \frac{(\mathbf{k}^2 + \mathbf{l}^2)\mathbf{N}^2 + \mathbf{m}^2\mathbf{f}^2}{\mathbf{k}^2 + \mathbf{l}^2 + \mathbf{m}^2} \quad \mathbf{f} < \omega < \mathbf{N}$$

Internal wave equations

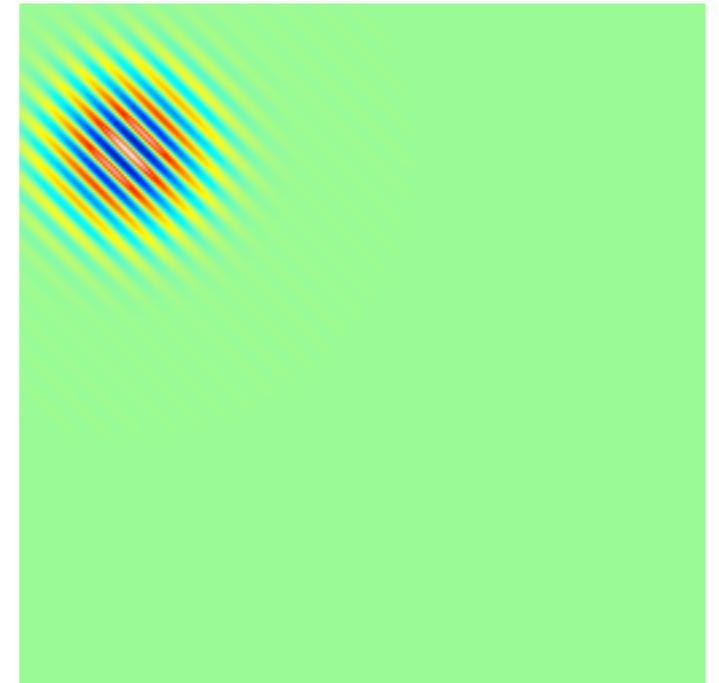
Vertically propagating waves

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Internal wave equations

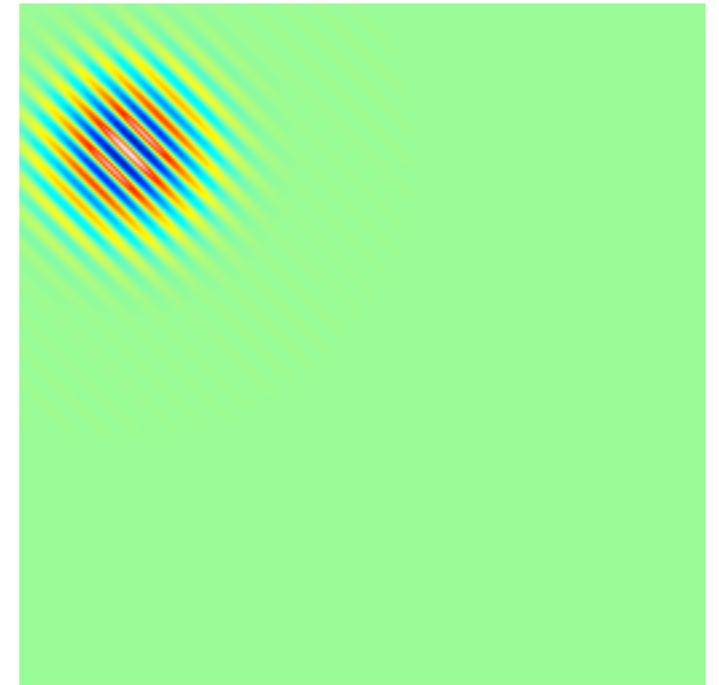
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(Glenn Flierl)

Internal wave equations

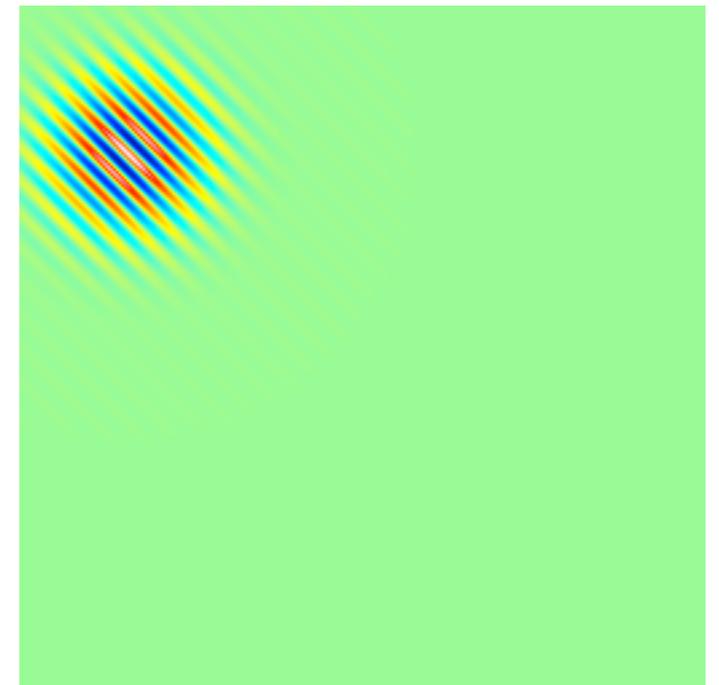
Vertically propagating waves

Try a solution of the form

$$\mathbf{u}(\mathbf{x}, \mathbf{y}, \mathbf{z}, t) = \hat{\mathbf{u}} e^{-i[\mathbf{k}\mathbf{x} + l\mathbf{y} + m\mathbf{z} - \omega t]}$$

Get polarization and dispersion relationships

$$\omega^2 = \frac{(\mathbf{k}^2 + l^2)N^2 + m^2 f^2}{\mathbf{k}^2 + l^2 + m^2} \quad f < \omega < N$$

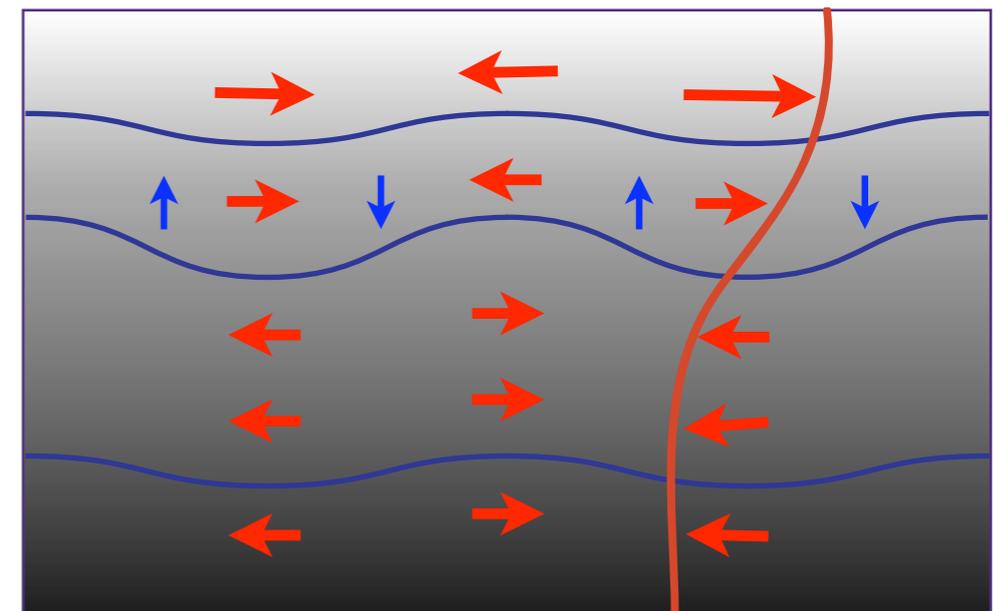


(Glenn Flierl)

Vertically standing waves

$$\mathbf{u}(\mathbf{x}, \mathbf{y}, \mathbf{z}, t) = \hat{\mathbf{u}} \Psi(\mathbf{z}) e^{-i[\mathbf{k}\mathbf{x} + l\mathbf{y} - \omega t]}$$

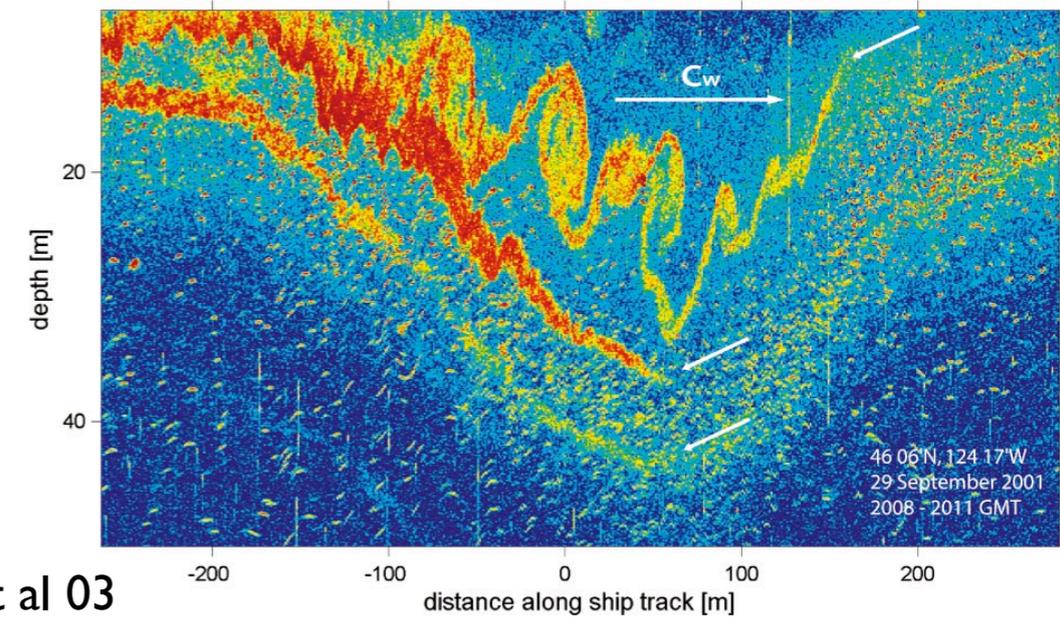
- rigid upper and lower boundaries
- similar to two-layer interfacial waves
- ocean stratification typically decreases with depth
- horizontal wavelength 10-100 km



Motivation - breaking waves mix the ocean

Waves break by shear instability
and/or convective overturning.

(see Staquet and Sommeria 04 for a review)

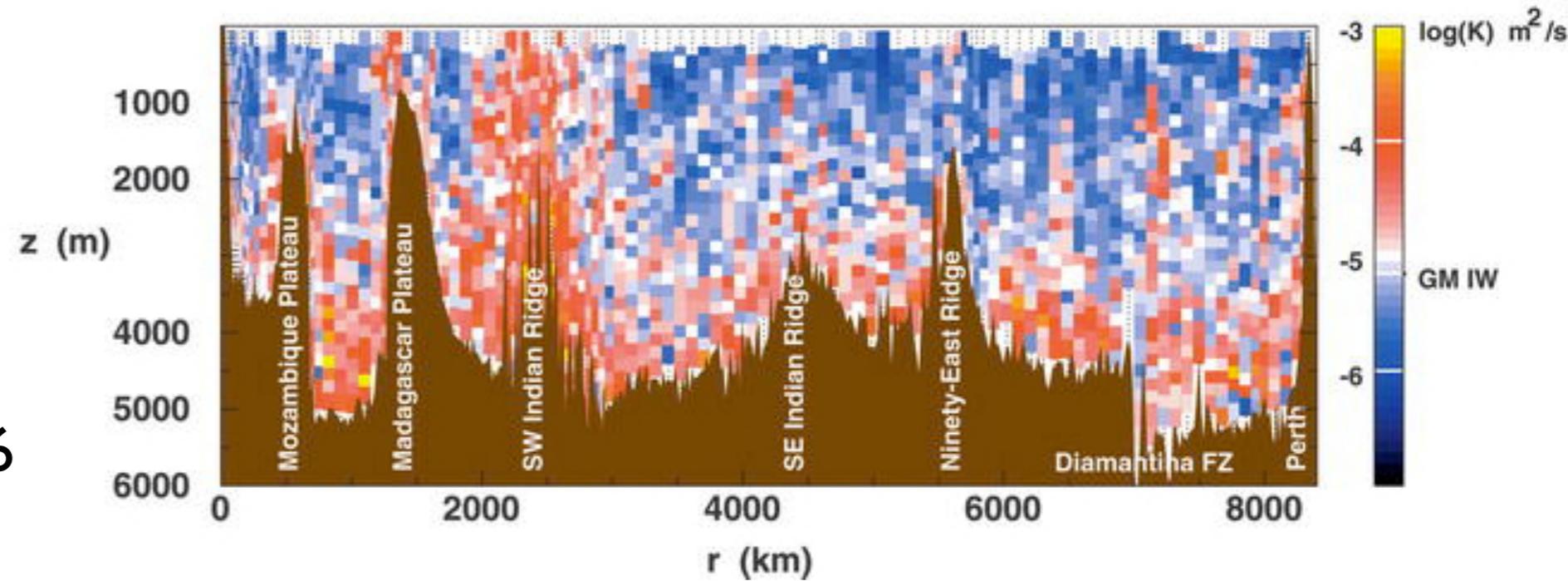


Moum et al 03

Patchiness of Deep Mixing matters

Spatial distribution of mixing reflects geography of internal wave generation, propagation and breaking

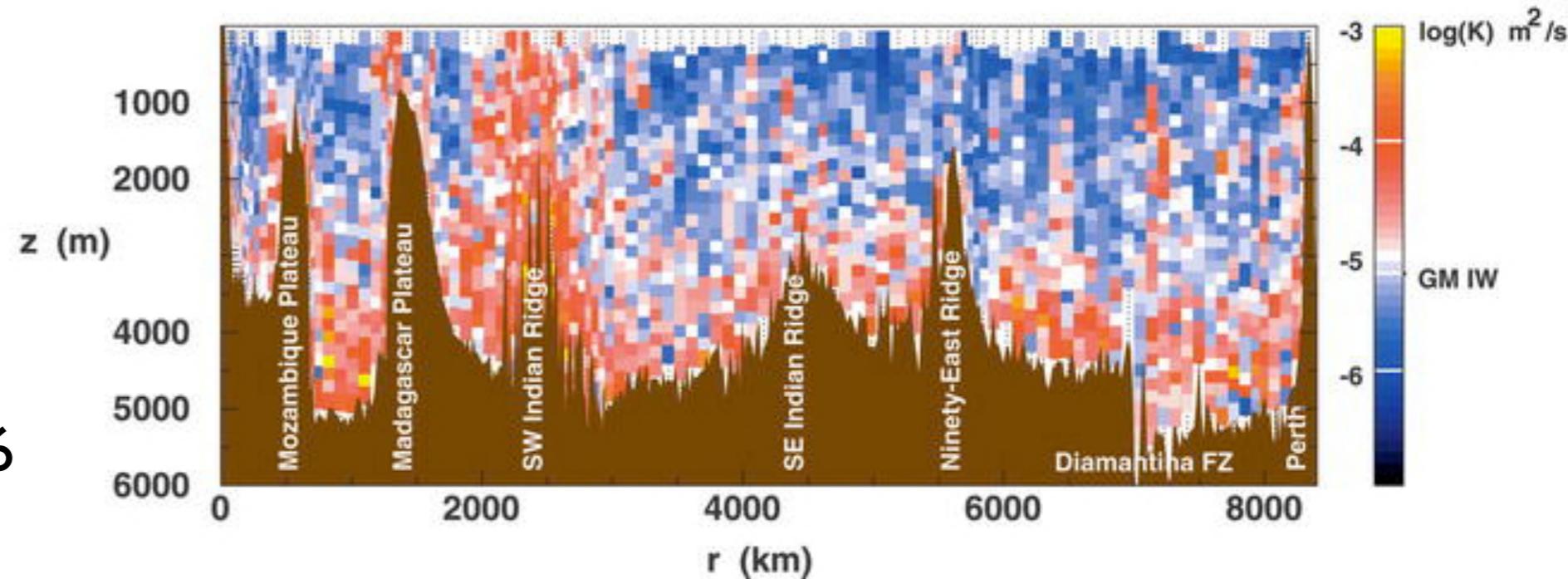
Kunze et al 06



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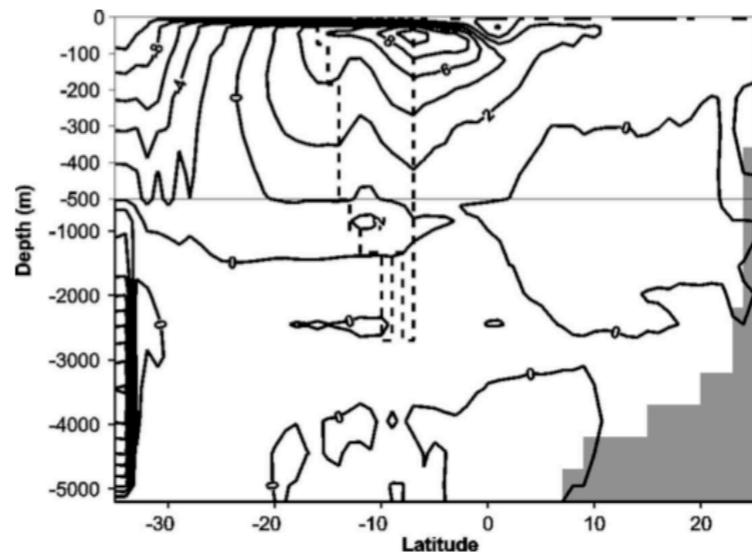


Palmer et al. (2007):

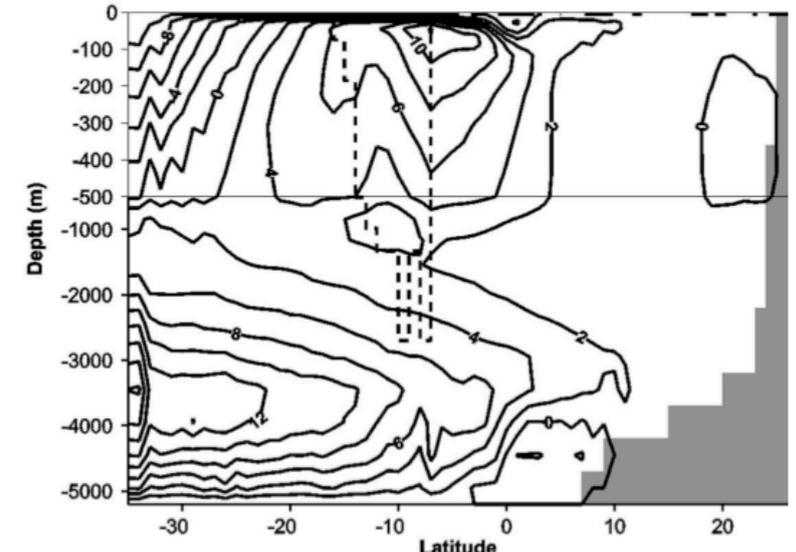
bottom-enhanced
diffusivity

=> deep overturning

Constant $\kappa = 1.2 \cdot 10^{-4}$



Bottom-enhanced diffusivity

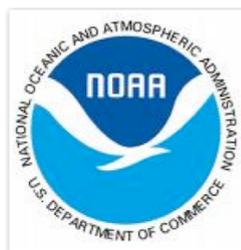


Cant' explicitly resolve internal waves in climate models.

Climate Process Team: Funded to combine theory, observations and simulations to develop and implement improved parameterizations of internal-wave driven mixing in global climate models

The Team:

Matthew Alford (UW)	Markus Jochum (NCAR)
Brian Arbic (U Michigan)	Jody Klymak (UVic)
Frank Bryan (NCAR)	Eric Kunze (Uvic)
Eric Chassignet (FSU)	William Large (NCAR)
Gokhan Danabasoglu (NCAR)	Sonya Legg (GFDL/Princeton)
Peter Gent (NCAR)	Jennifer MacKinnon (SIO)
Mike Gregg (UW)	Rob Pinkel (SIO)
Steve Griffies (GFDL)	Kurt Polzin (WHOI)
Robert Hallberg (GFDL)	Harper Simmons (UAF)
Steve Jayne (WHOI)	Lou St. Laurent (WHOI)

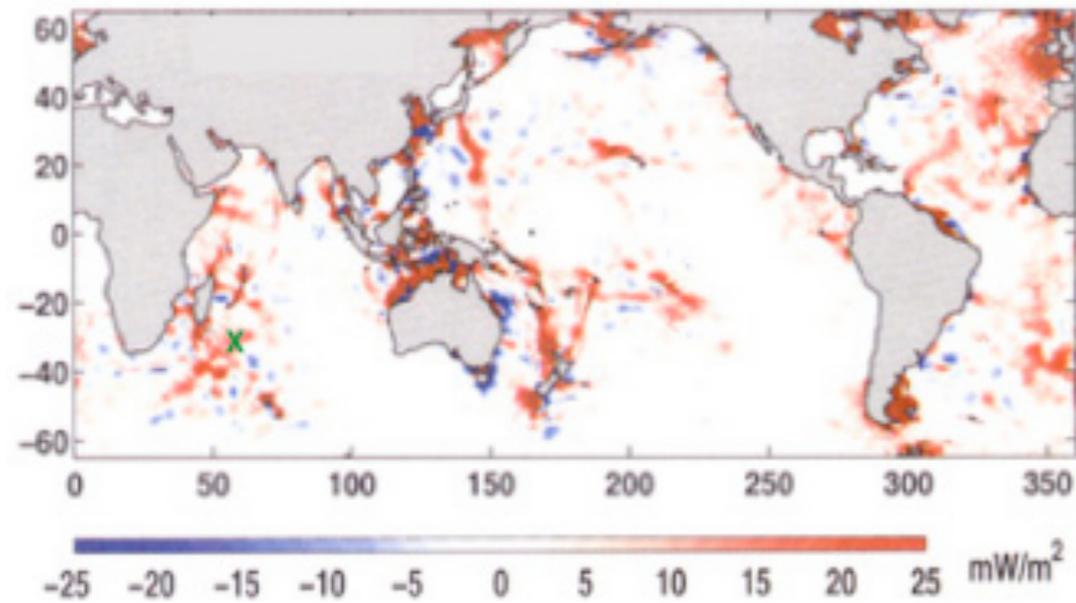


Cant' explicitly resolve internal waves in climate models.

3 steps to parameterize their role:

I) Wave generation

Internal-Tide Generation

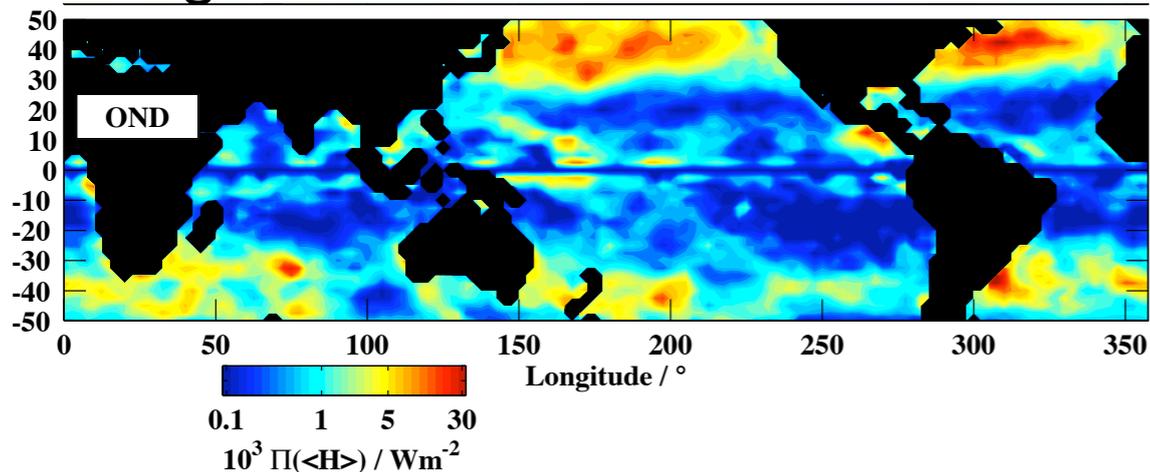


Egbert and Ray 01

Generation where barotropic (astronomical) tides are large and topography is rough. Fairly steady in time.

[St. Laurent, Nycander talks]

Wind-generated near-inertial internal waves



Alford 01

Generation by rotating component of wind stress, mirrors storm tracks. NOT steady in time (episodic storms + seasonal cycle), but seasonally averaged pattern reasonably(?) consistent

[Simmons talk]

Cant' explicitly resolve internal waves in climate models.

3 steps to parameterize their role:

1) Wave generation

2) Some waves break “locally”, leading to a global pattern of mixing that mirrors wave generation. This can be roughly broken down into weakly and strongly nonlinear dynamics

A: wave action = E/ω

$$\frac{\partial A}{\partial t} + \underbrace{\nabla_{\mathbf{r}} \cdot (\mathbf{C}_g + \bar{\mathbf{u}})}_{\text{flux divergence}} A + \underbrace{\nabla_{\mathbf{p}} \cdot \mathcal{R} A}_{\substack{\text{refraction by} \\ \text{mesoscale} \\ \text{energy transfer} \\ \text{to other} \\ \text{internal waves}}} = \underbrace{T_r + S_o - S_i}_{\substack{\text{sources and} \\ \text{sinks}}}$$

**Primary
wave**

Cant' explicitly resolve internal waves in climate models.

3 steps to parameterize their role:

1) Wave generation

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$$\frac{\partial A}{\partial t} + \nabla_{\mathbf{r}} \cdot (\mathbf{C}_g + \bar{\mathbf{u}})A + \nabla_{\mathbf{p}} \cdot \mathcal{R}A = T_r + S_o - S_i$$

flux divergence

refraction by mesoscale

energy transfer to other internal waves

sources and sinks

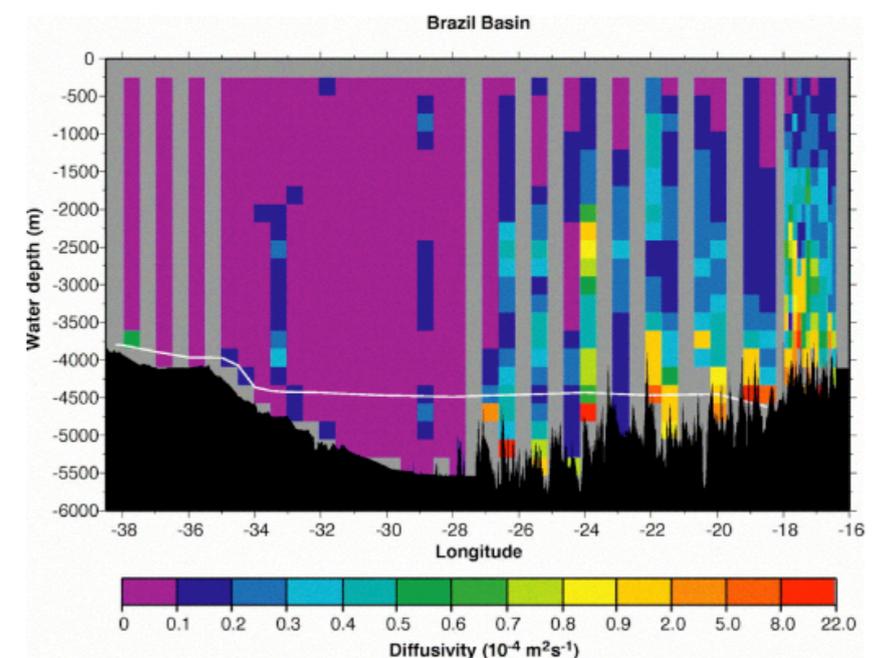
Primary wave

[polzin 04]

As a wave propagates upwards from a source region, it steadily loses energy to other internal waves (+mesoscale effects), which in turn lose energy to other waves... which dissipate locally. So an internal tide leaves behind a wake of dissipation

Secondary (smaller scale?) waves

$$T_r = \epsilon$$



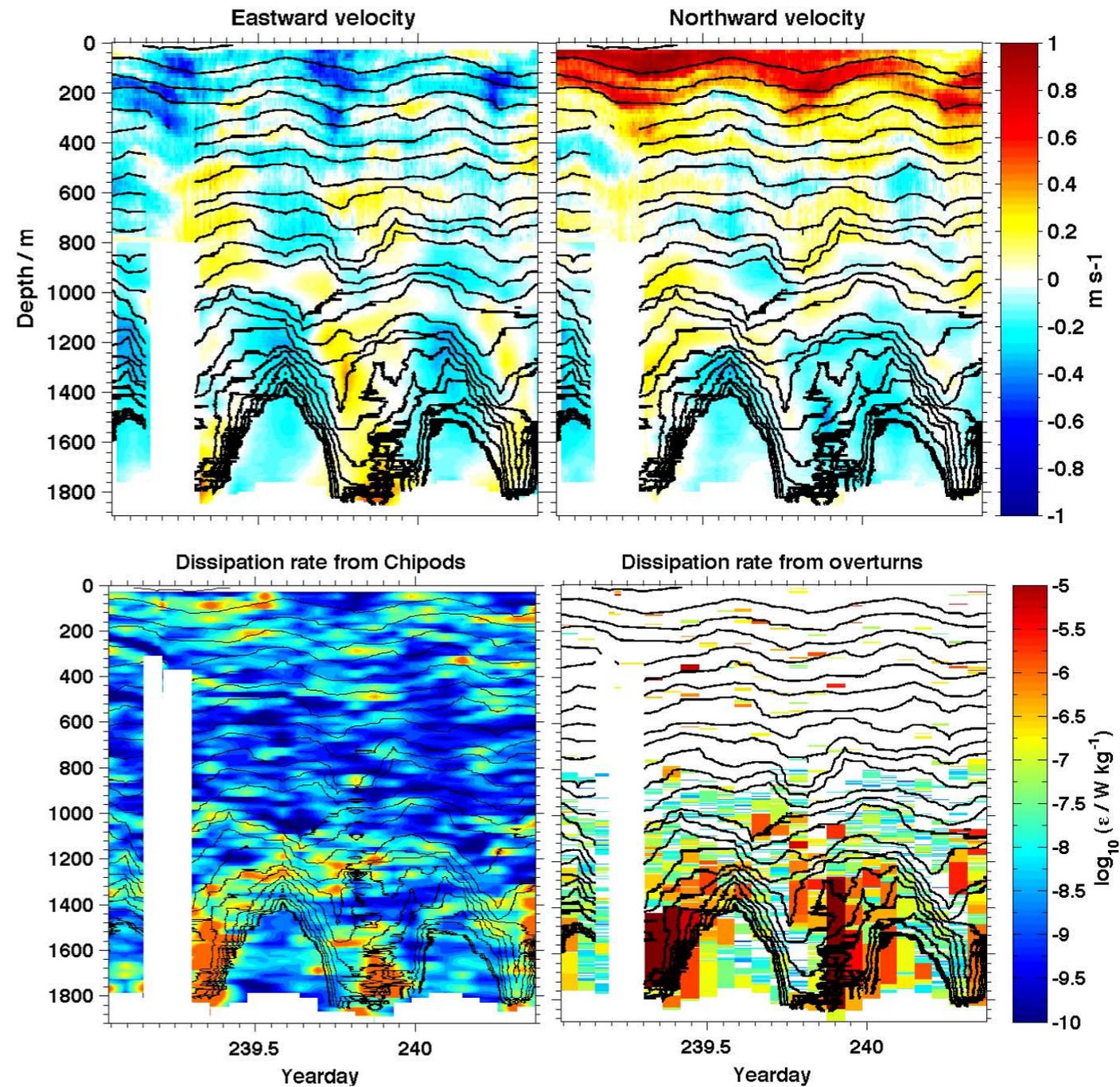
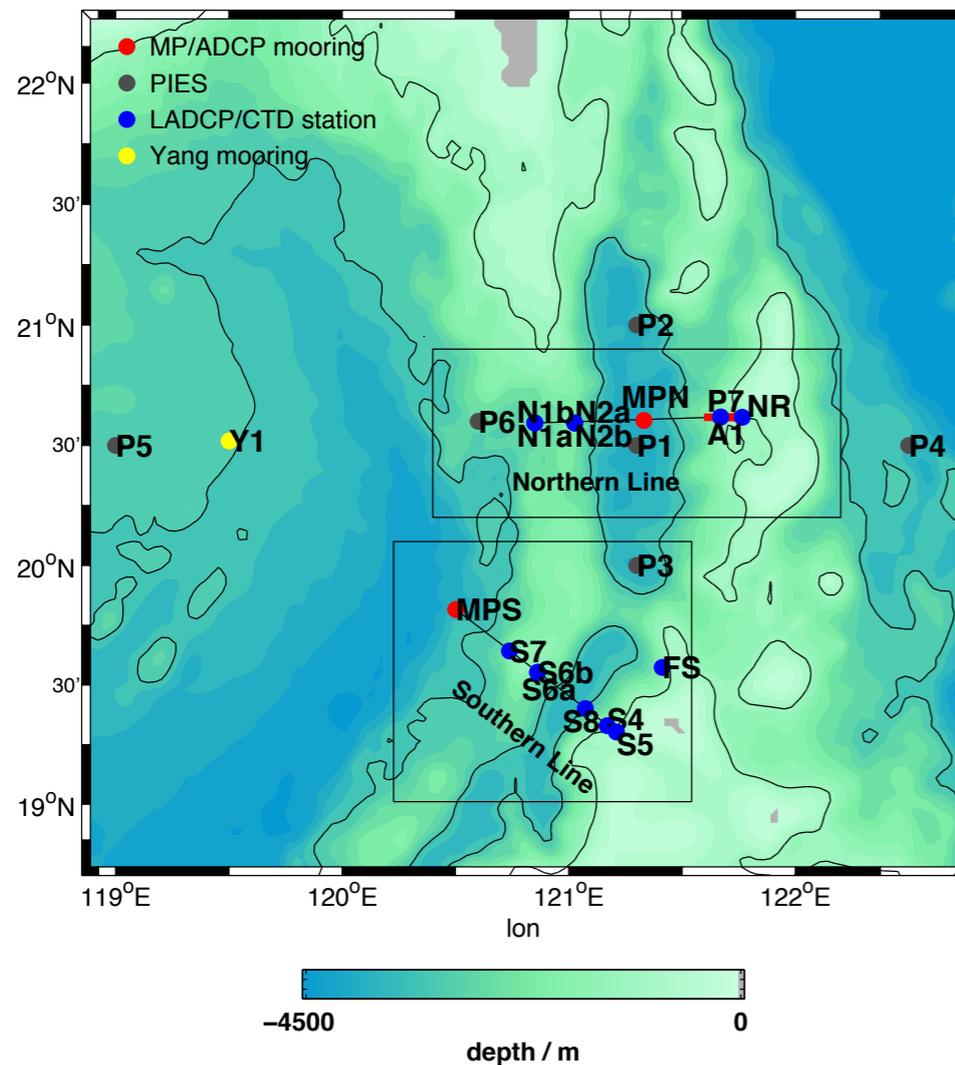
Polzin et al. 97

Nearfield: strongly nonlinear wave breaking

Internal hydraulic jumps, strong bores, solitons etc

[Akylas, Carr, Farmer, Helfrich, Shroyer, Staquet, Weidman talks]

Strong tidally driven overturning in the South China Sea [Alford, Simmons, Nash, MacKinnon, Klymak, Pinkel]



(Alford talk on Wednesday)

“Farfield” wave breaking / mixing

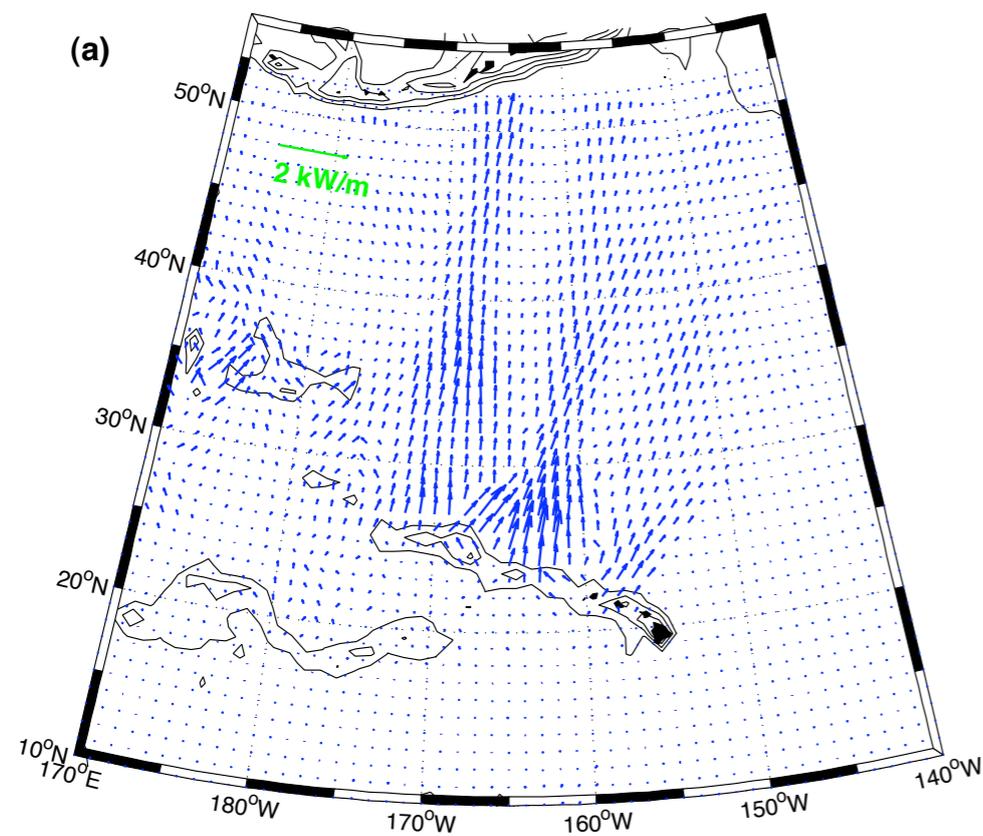
Most (70-90%) internal tide energy escapes to propagate thousands of km away.

Where do these waves break? [St. Laurent and Nash 04]

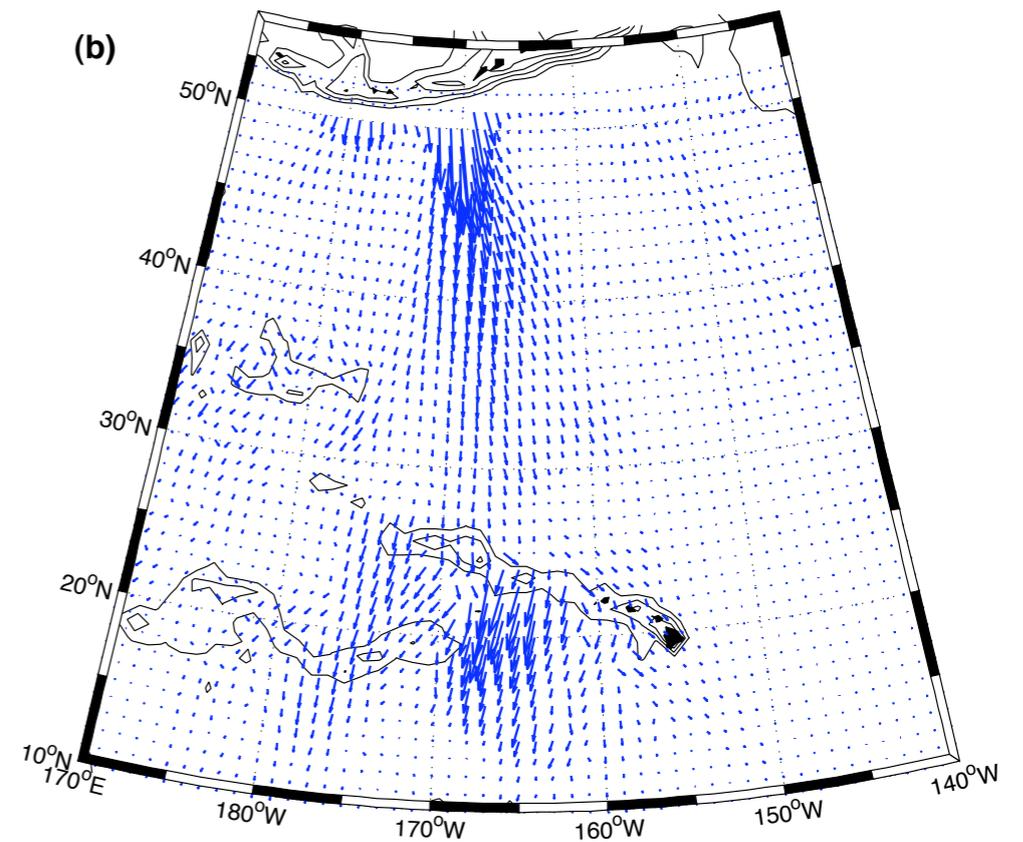
Altimetric tidal fluxes

Zhao and Alford

Northbound



Southbound



The waves that get away...

- Energy lost to strong wave-wave interactions like Parametric Subharmonic Instability (PSI)
- Propagating low-mode waves lose some energy through a slow bleed to ambient internal wave field through wave-wave interactions, supplying the background dissipation rate.
- Some energy lost through interaction with mesoscale, trapping in regions of high vorticity (e.g. Kuroshio) or high shear (e.g. equatorial undercurrent)
- Whatever isn't lost in transit “breaks” when waves crash into continental slope or other topography

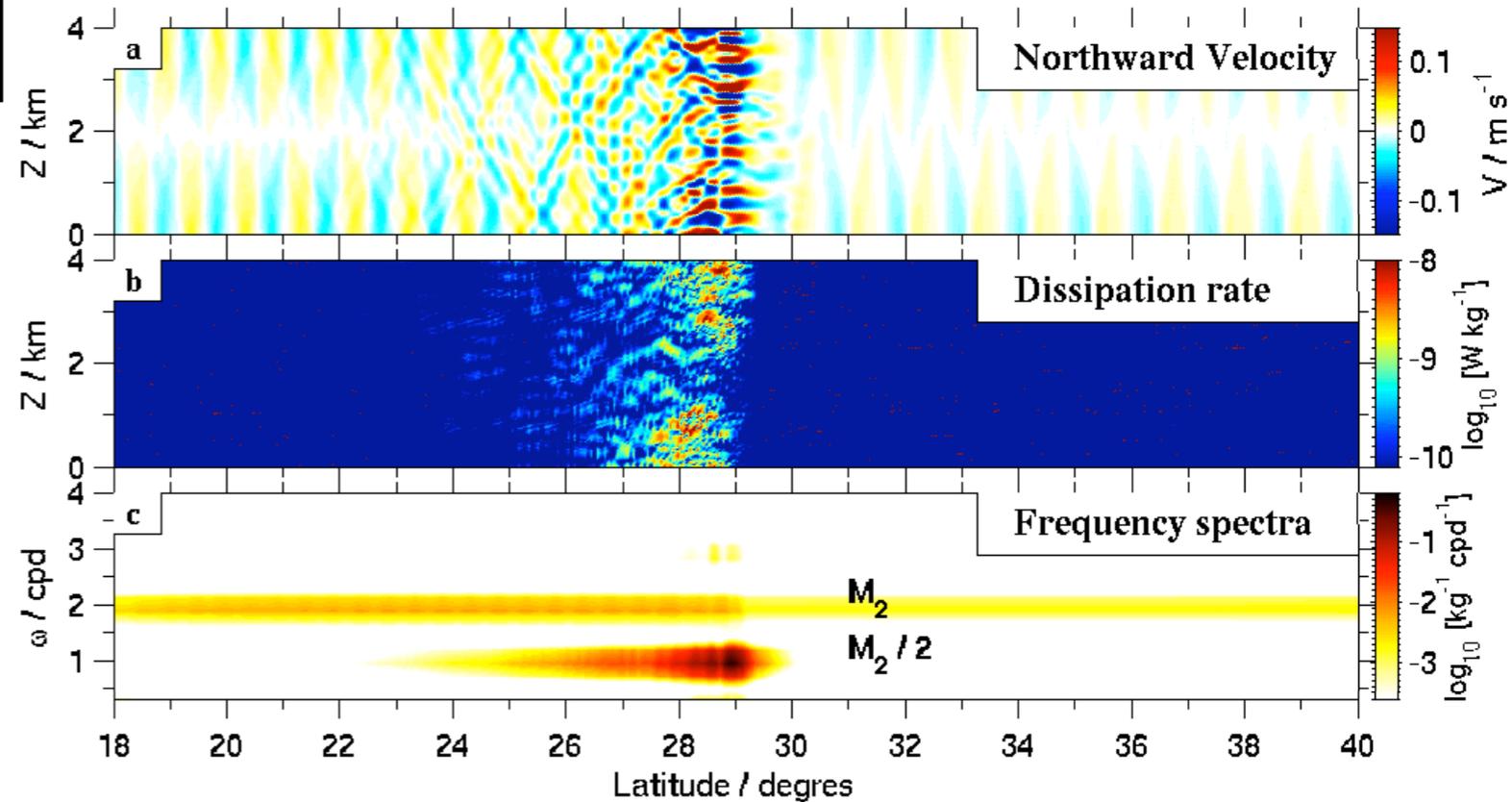
How important is each?

Parametric Subharmonic Instability

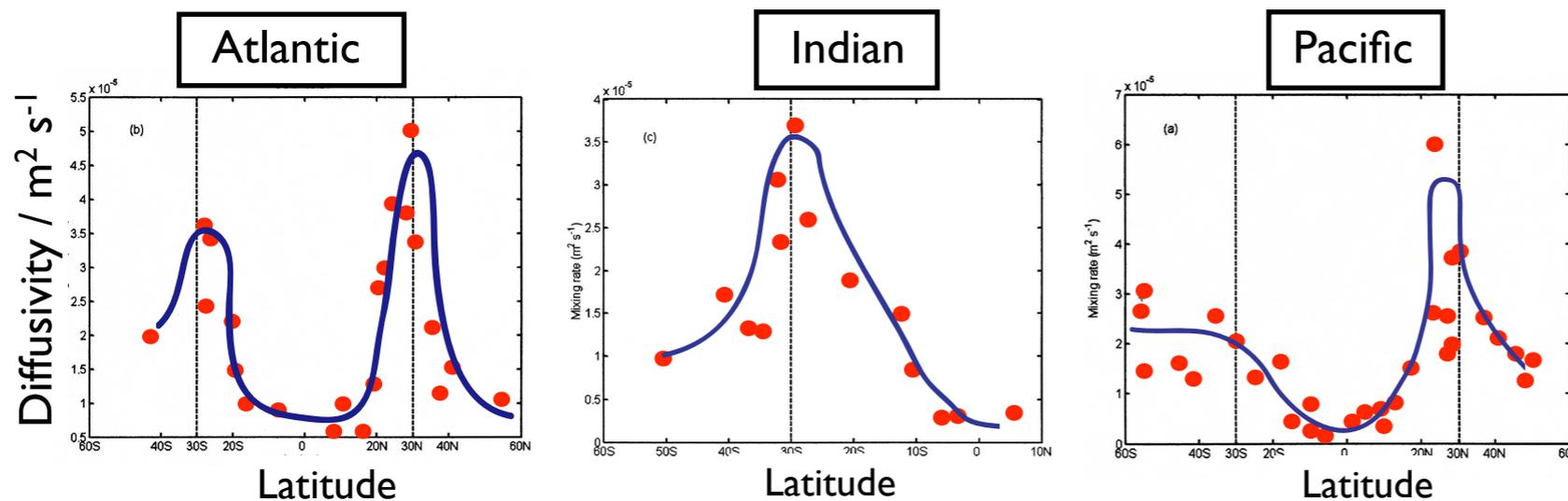
Convergence of numerical internal tide fluxes (MacKinnon and Winters et al 05)

Parametric Subharmonic Instability (PSI)

- Strong resonant instability near 29° where $f = M_2/2$
- Energy loss from M_2 internal tide to small-scale $M_2/2$ motions
- Subharmonic waves have equal amounts of upward and downward propagating energy



Convergence of altimetric internal tide fluxes (Tian et al 06)

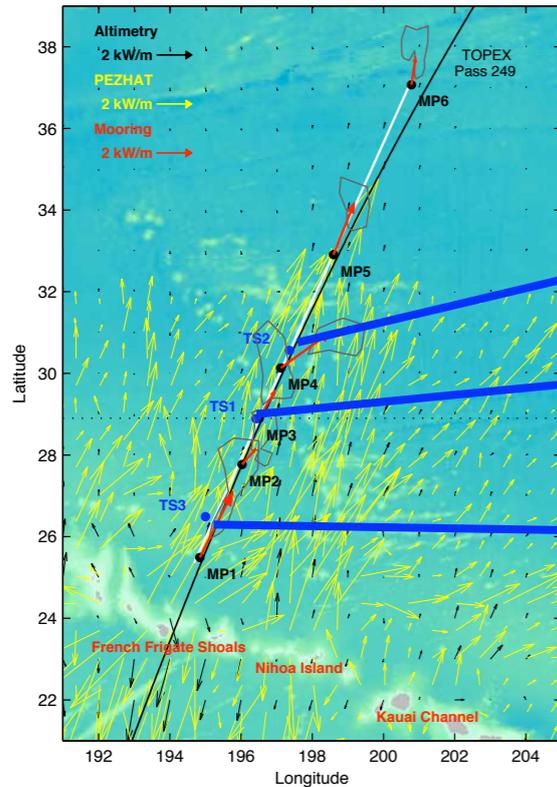


Hibiya and Nagasawa 04
 MacKinnon and Winters 05
 Furuichi et al 05
 Young, Tsang and Balmforth 07
 Hibiya et al 07
 Young et al 08
 Munroe talk

Observations of PSI

Internal Waves Across the Pacific

Alford, MacKinnon, Pinkel, Klymak, Peacock

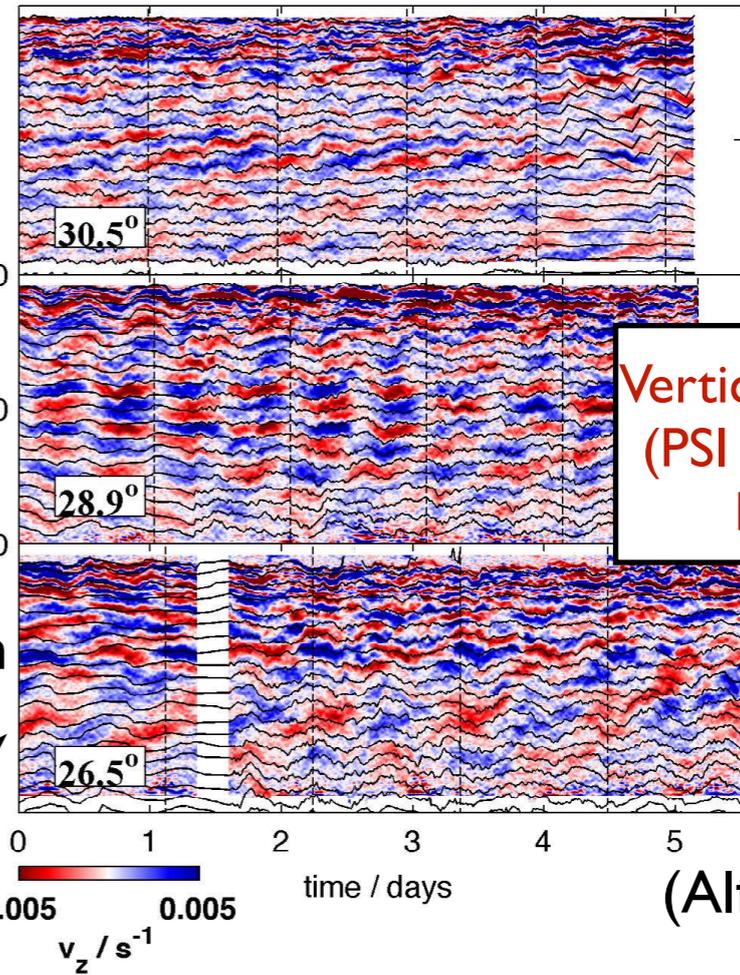


Surface generation

Phase \uparrow Energy \downarrow

Surface generation

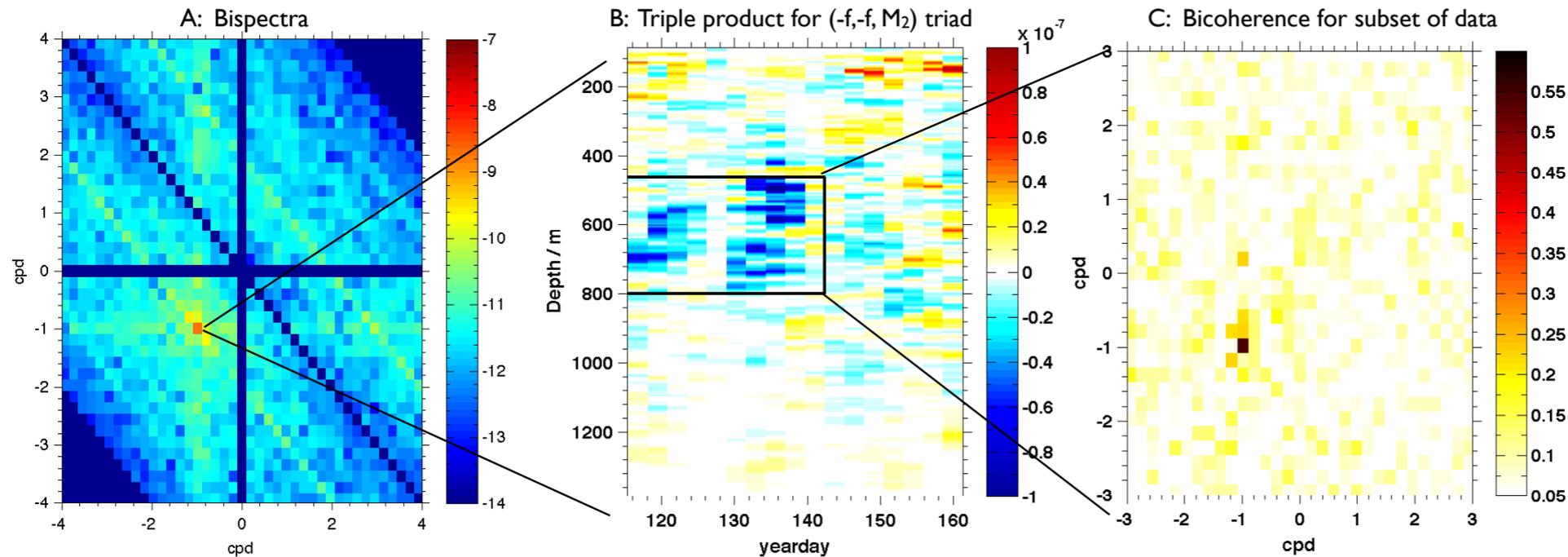
Phase \uparrow Energy \downarrow



Vertically standing
(PSI generation)
 $Ri = 0.7$

(Alford et al 07)

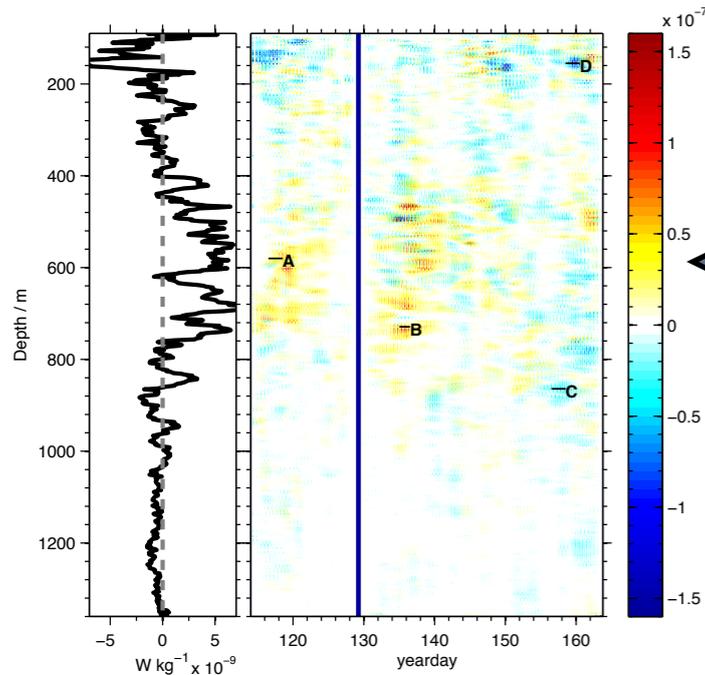
Bispectra show significant phase-locked energy transfer from tidal to subharmonic motions



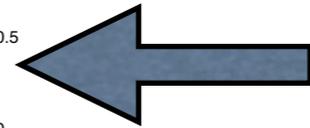
(MacKinnon et al 11a,b)

IWAP continued

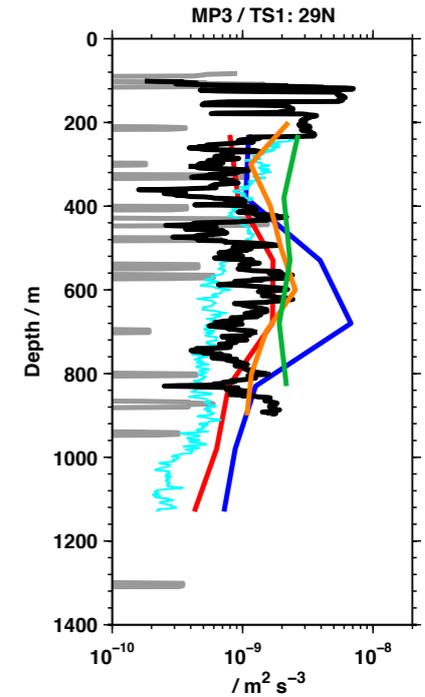
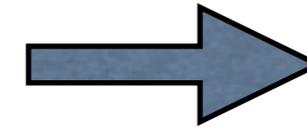
However, net transfer rate and local dissipation rate are quite modest



Rate of energy transfer to subharmonic

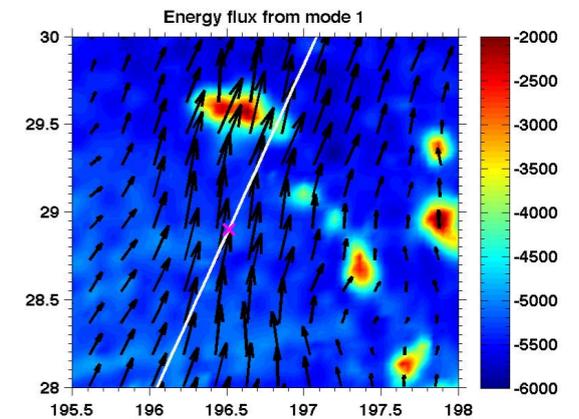


Local dissipation rate

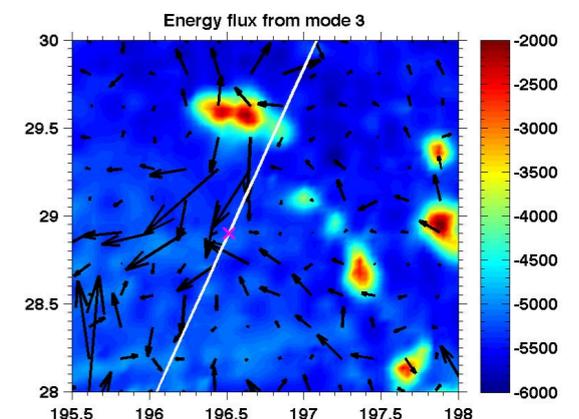


Discrepancy between observations and predicted catastrophe likely due to

- Multiple tidal modes going various directions
- Internal tide changes in time, due to both mesoscale refraction and S-N cycle, both of which limit PSI growth rates (Hazewinkel and Winters II)



mode-1



mode-3

The waves that get away...

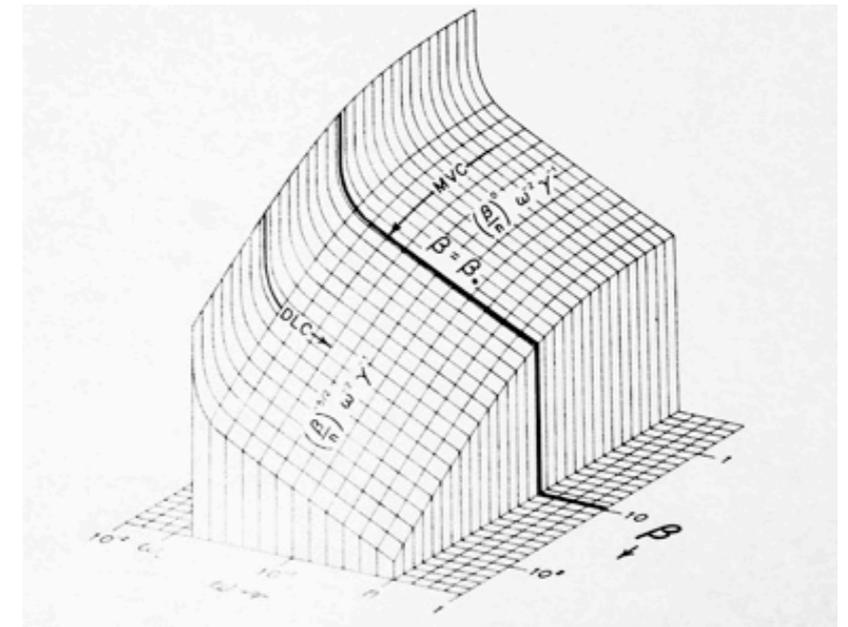
- Some energy lost to PSI, but probably not much (~10-20% in a few specific places...?)
- Propagating low-mode waves lose some energy through a slow bleed to ambient internal wave field through wave-wave interactions, supplying the background dissipation rate.
- Some energy lost through interaction with mesoscale, trapping in regions of high vorticity (e.g. Kuroshio) or high shear (e.g. equatorial undercurrent)
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How important is each?

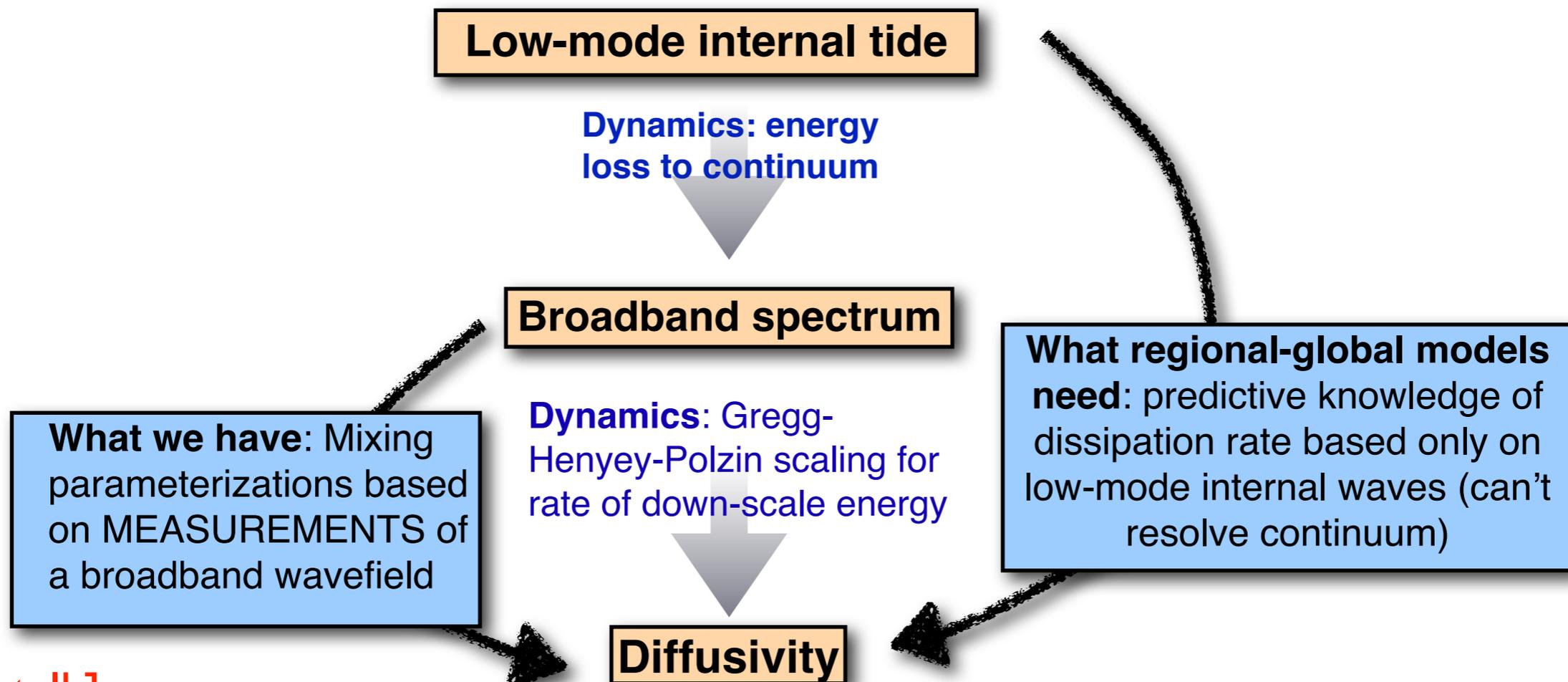
Fate of internal tides II: nonlinear wave-wave interactions

Empirical observations show a remarkably universal distribution of energy in wavenumber and frequency space, with energy spectra in both directions tending towards -2 slopes

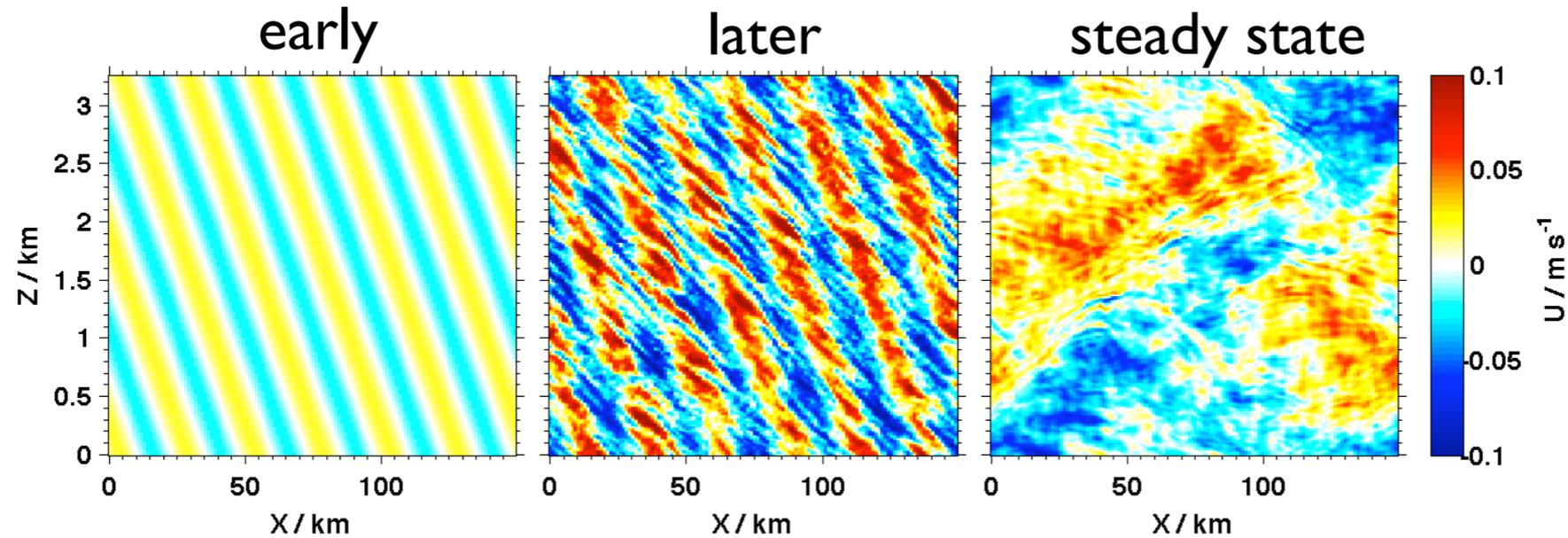
(McComas and Muller 81, Muller et al 86, Lvov and Tabak 01, Lvov and Tabak 04, Lvov et al 04, Polzin 04 (x 2))



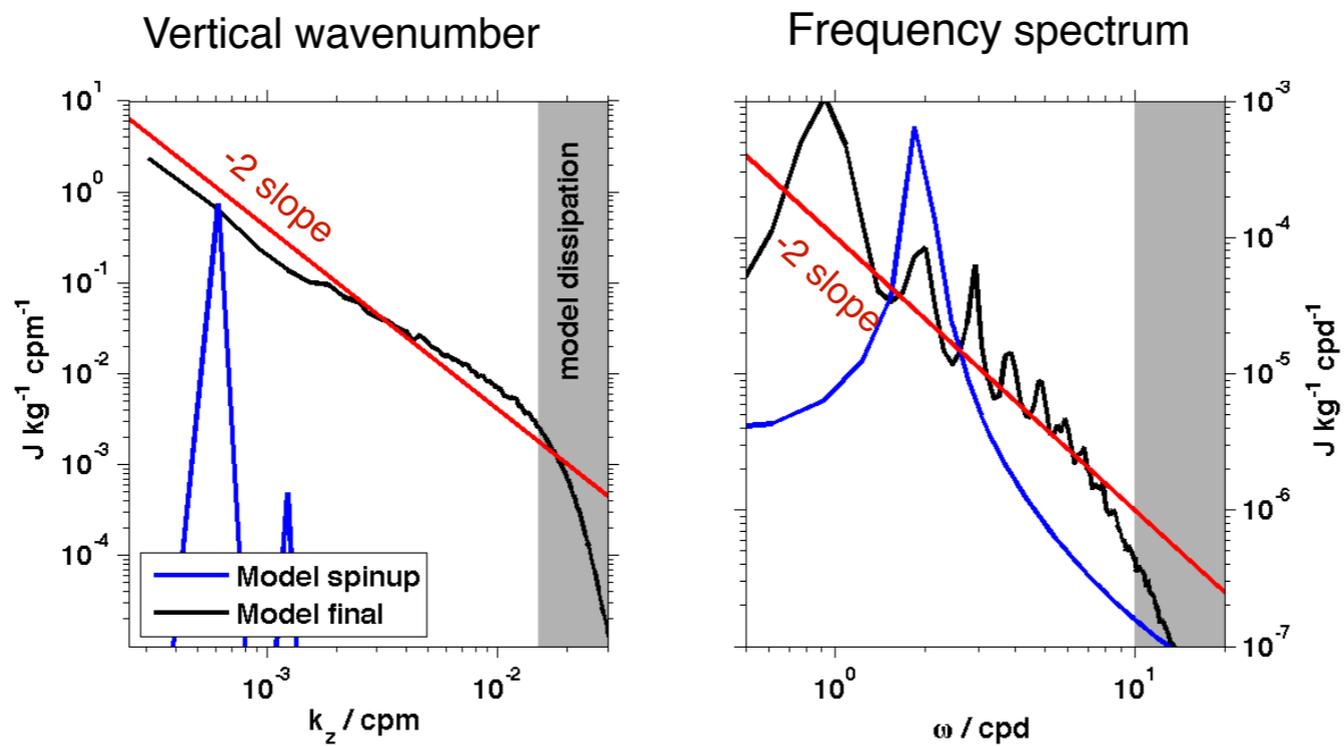
(Garrett and Munk 1975)



Numerical studies of energy transfer



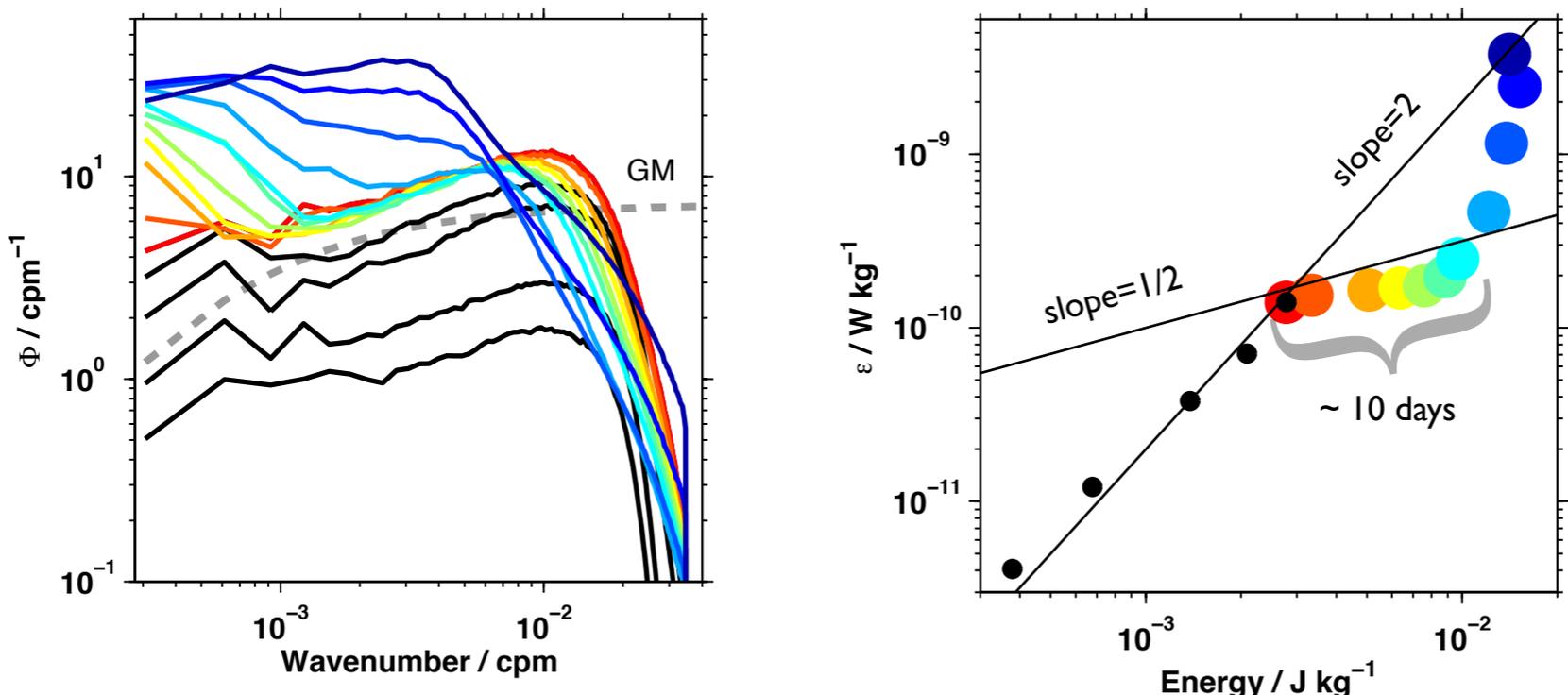
Energy flows to smaller scales, till a broadband spectrum is created in which continued tidal forcing is balanced by down-scale energy transfer and dissipation.



[Mackinnon, in prep. Also Winters and D'Asaro 97, Hibiya et al 02, Lvov and Yokoyama 09]

Question: Empirical results (MacKinnon and Gregg, 02,05) show that the relationship between low-mode shear and turbulent dissipation scales differently in the open ocean and shallow water. Why? MG suggest that either the lower vertical bandwidth or the higher tidal variability in coastal water might be to blame

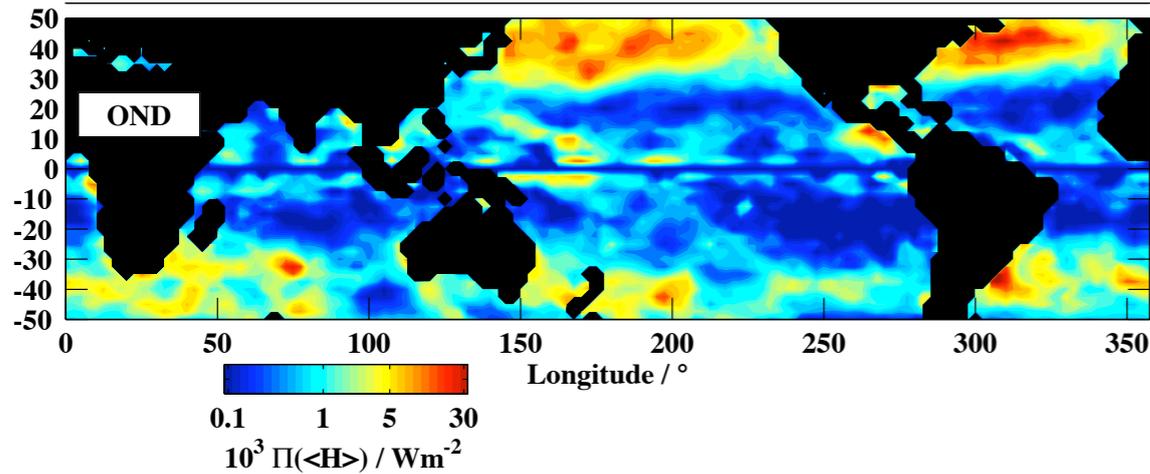
Changing the forcing: Wavenumber spectra (left) and dissipation rate (right) are shown for 5 simulations run to steady state with different levels of wave forcing (black lines/dots). For the fifth simulation, the forcing tide is abruptly changed, and the evolution over 20 days is shown in color (red to blue).



[MacKinnon, in prep]

- Steady state: The dissipation rate scales quadratically with the spectral level ($\epsilon \sim E^2$), in agreement with numerous previous theories and simulations (e.g. Winters and D'Asaro)
- Variable tide: When the forcing is changed, it takes time (~ 20 days here) for the small-scale waves to adjust to the new equilibrium. Initially, dissipation increases with increasing tidal energy, but more slowly ($\epsilon \sim E^{1/2}$). This is the same scaling observed by MacKinnon and Gregg, perhaps because in coastal areas internal tides are highly variable and the wavefield is never in steady state.

Near-Inertial wave generation and decay

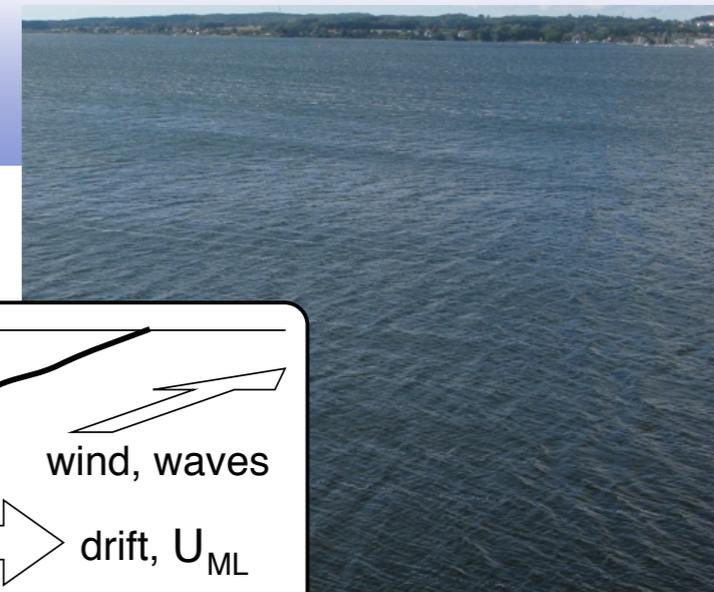


[Alford 01,03, Plueddemann and Farrar]

How and where do wind-generated near-inertial waves break?

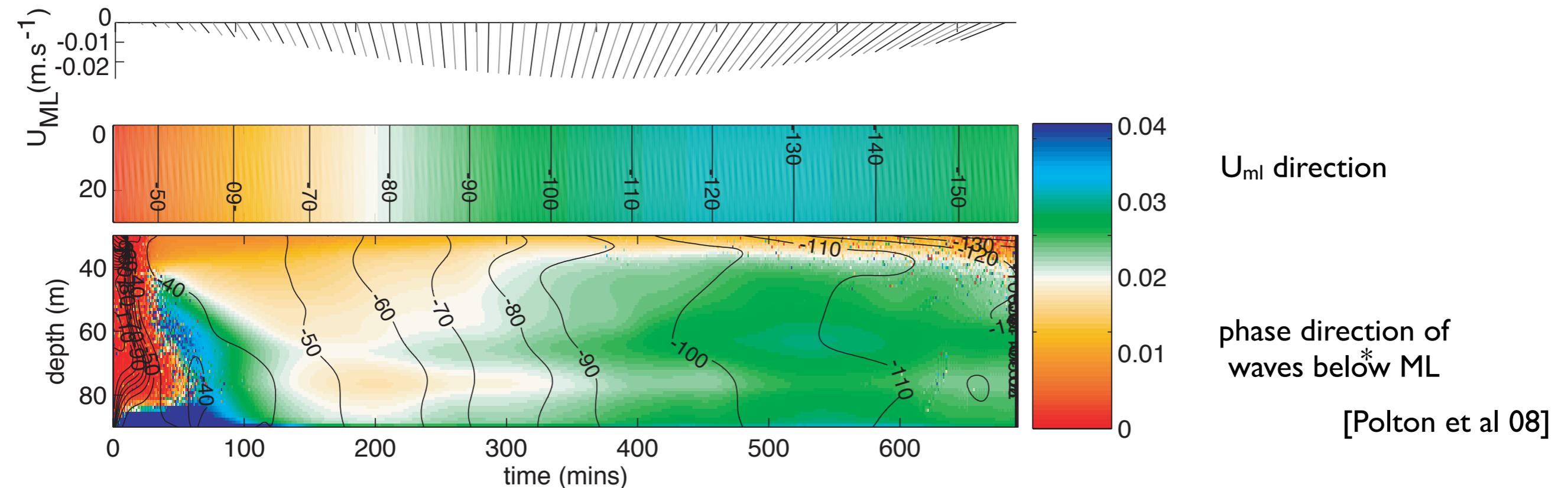
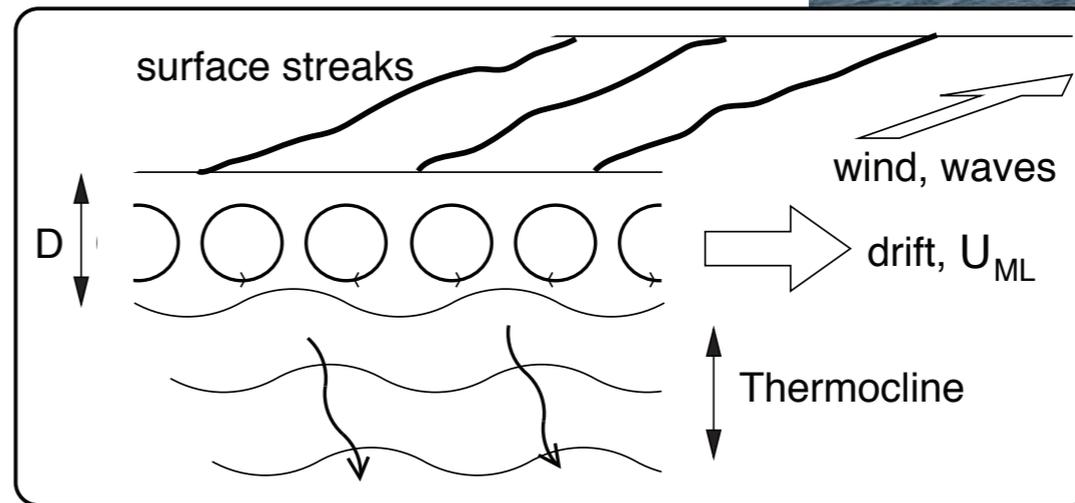
- What sorts of waves are generated? Low mode? High mode? Do we know?
- How does interaction with mesoscale upper ocean vorticity (eddies etc) affect generation and propagation? (Young and Ben Jelloul 97, Balmforth et al 98, Balmforth and Young 99, Klein and Llewellyn Smith 01, Klein et al 04, Danioux et al 08, Elipot and Lumpkin 09)
- What percentage of near-inertial energy dissipates “locally”?
- What happens to the low-mode near inertial waves? Clearly seen in the deep ocean (Alford and Whitmont 07), but we don't know how far they get (Simmons talk on xx)

High-frequency waves in the upper ocean



Langmuir-cells create high-frequency (~10 min) internal waves that extract energy from the ML and break to produce mixing in the stratified layer below

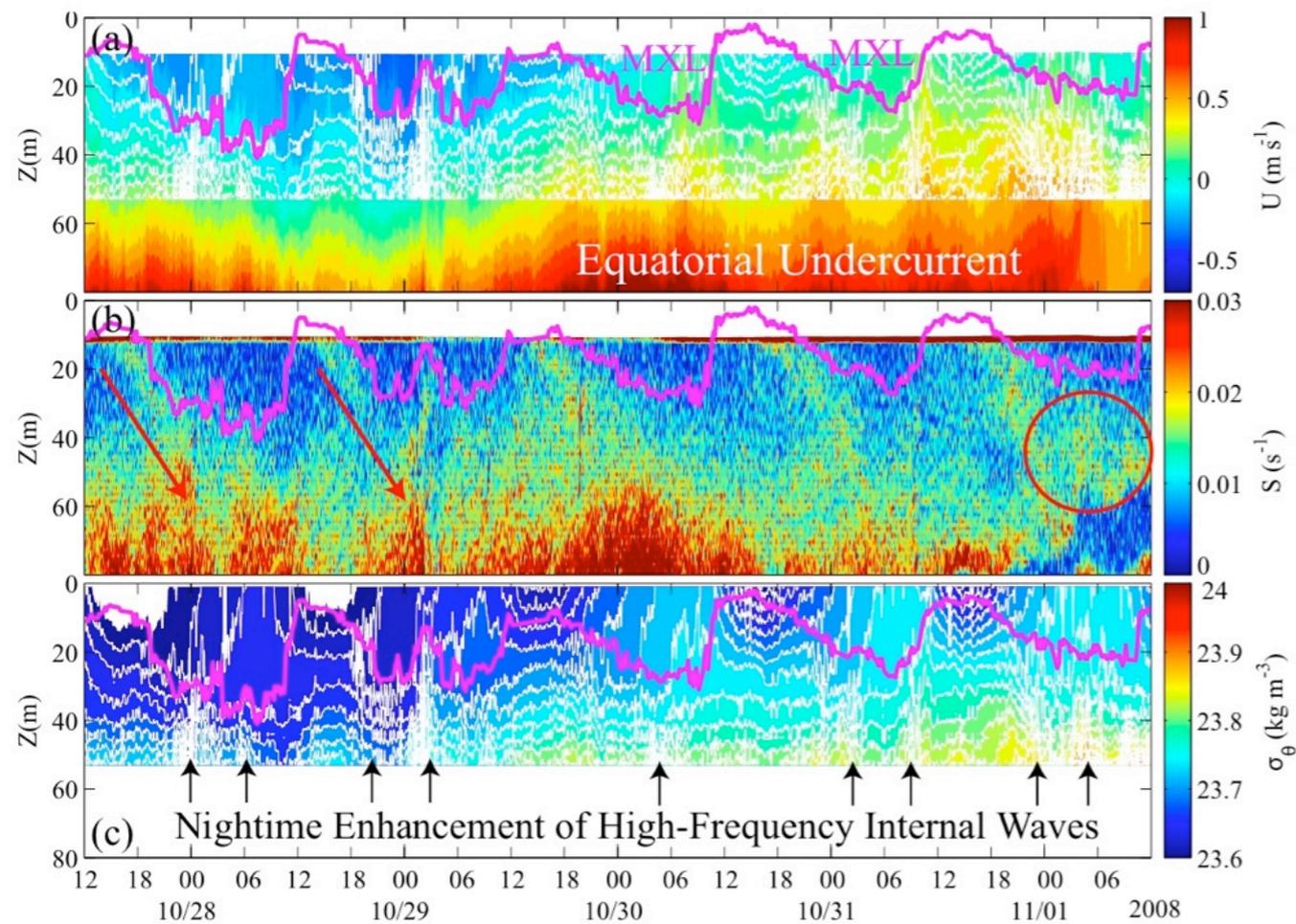
$$c_p = \frac{\omega}{k} = U_{ml}$$



These high-frequency waves may be trapped (and dissipate) in high-N transition layer separating upper ocean from main thermocline

High-frequency waves generated by convection

Similar high-frequency waves generated by convection banging on the base of the ML.



Data from equatorial Pacific, 0° N 140° W [R.-C. Lien]

Similar equatorial measurements

[McPhaden and Peters 1992; Moum et al. 1992, Gregg et al. 1985; Wijesekera and Dillon 1991, Moum et al 09]

Other studies of high-frequency internal waves generated by “turbulent” motions:

Dohan and Sutherland 05,
Taylor and Sarker 07

Diamessis talk

Conclusions

- Two species of internal waves dominate the energy (and energy flux) of the oceanic internal wave band - internal tides and wind-forced near-inertial waves.
- In both cases, most of the energy radiates away from the generation site. Candidate dissipation mechanisms include topographic scattering, wave-wave interactions (PSI), interaction with the mesoscale...??
- Other types of waves (lee waves, high-frequency waves from turbulent motions) may be important parts of local energy and mixing budgets
- Mixing due to wave breaking is patchy in both space and time. Understanding and predicting these patterns is important for proper modeling of heat and other tracer distributions.

Open questions

- Near-inertial waves in the ocean? What kinds of waves does the wind make? How far do they go? How do they break?
- How do internal waves interact with the mesoscale? Some energy lost through interaction with mesoscale, trapping in regions of high vorticity (e.g. Kuroshio) or high shear (e.g. equatorial undercurrent) [[Buhler, Williams talks](#)]
- Seems likely that much of the propagating low-mode energy is dissipated where waves scatter off topography [[Holmes-Cerfon, Kelly](#)] or crash into the continental shelf or other large topography. Do we understand this process? [Eriksen 82, Johnston et al 03, Johnston and Merrifield 03, Legg and Adcroft 03, Nash et al 04, Martini et al 11, Kelly, et al 11, Klymak et al 11]

