Equilibration of the Southern Ocean Stratification With and Without Topography

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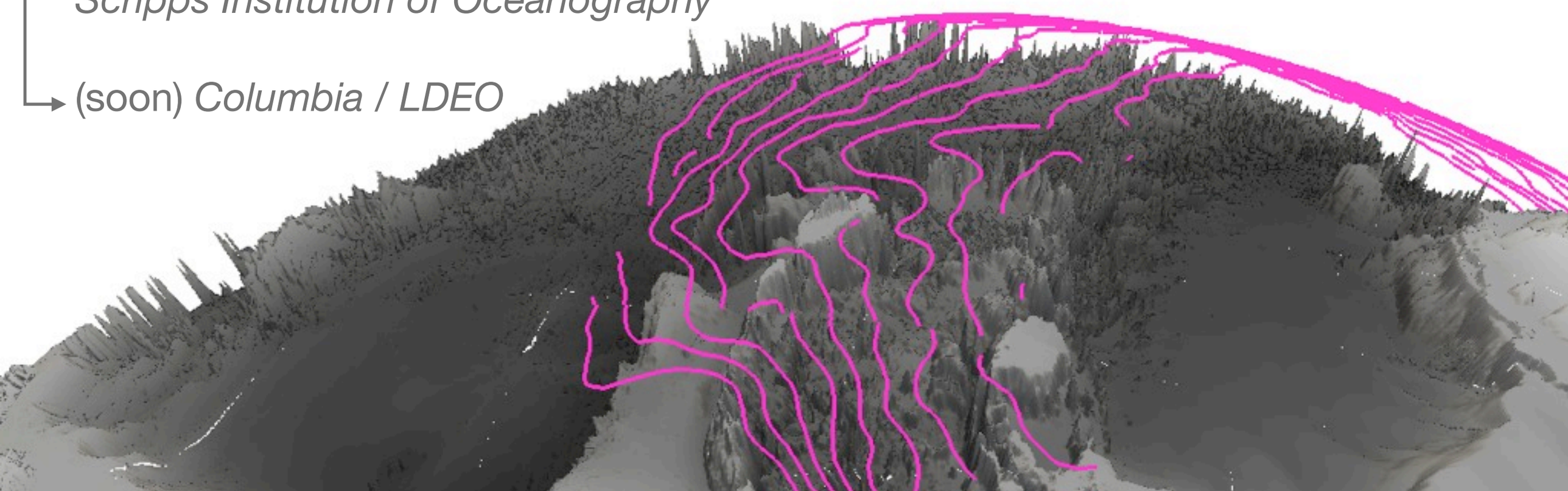
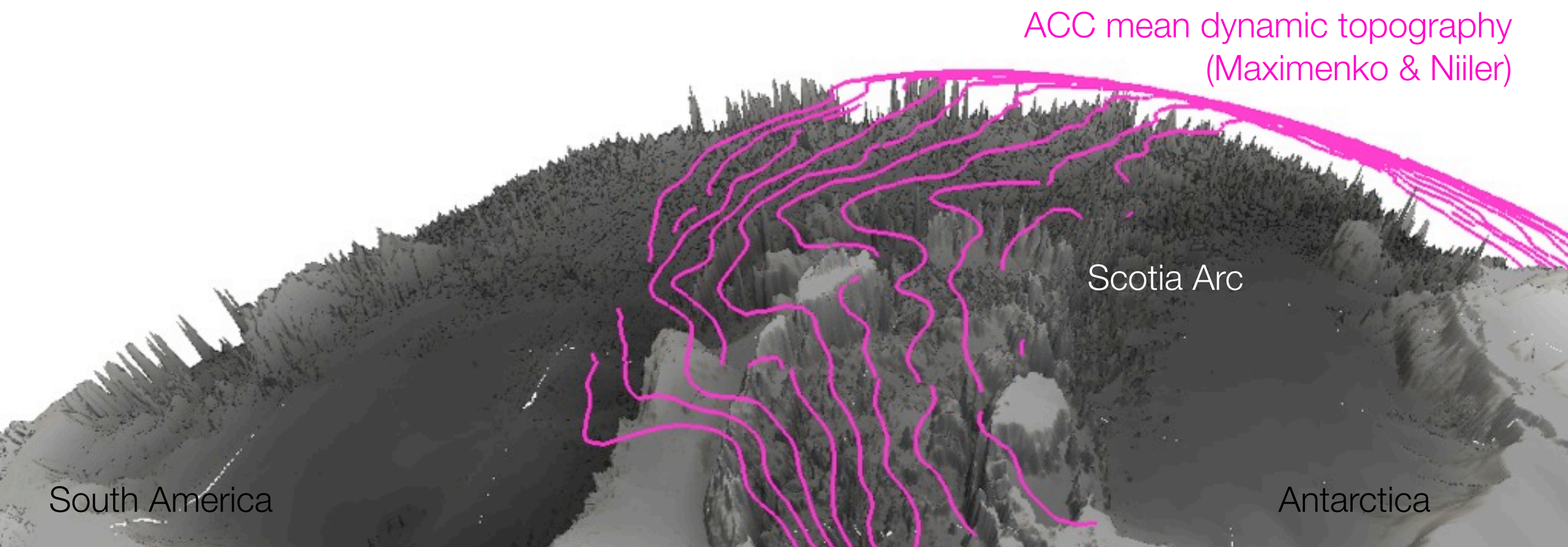
(soon) *Columbia / LDEO*




ACC mean dynamic topography
(Maximenko & Niiler)
Scotia Arc
South America
Antarctica

Eddy Saturation

Many model studies have investigated the response of the Southern Ocean to changing wind stress:

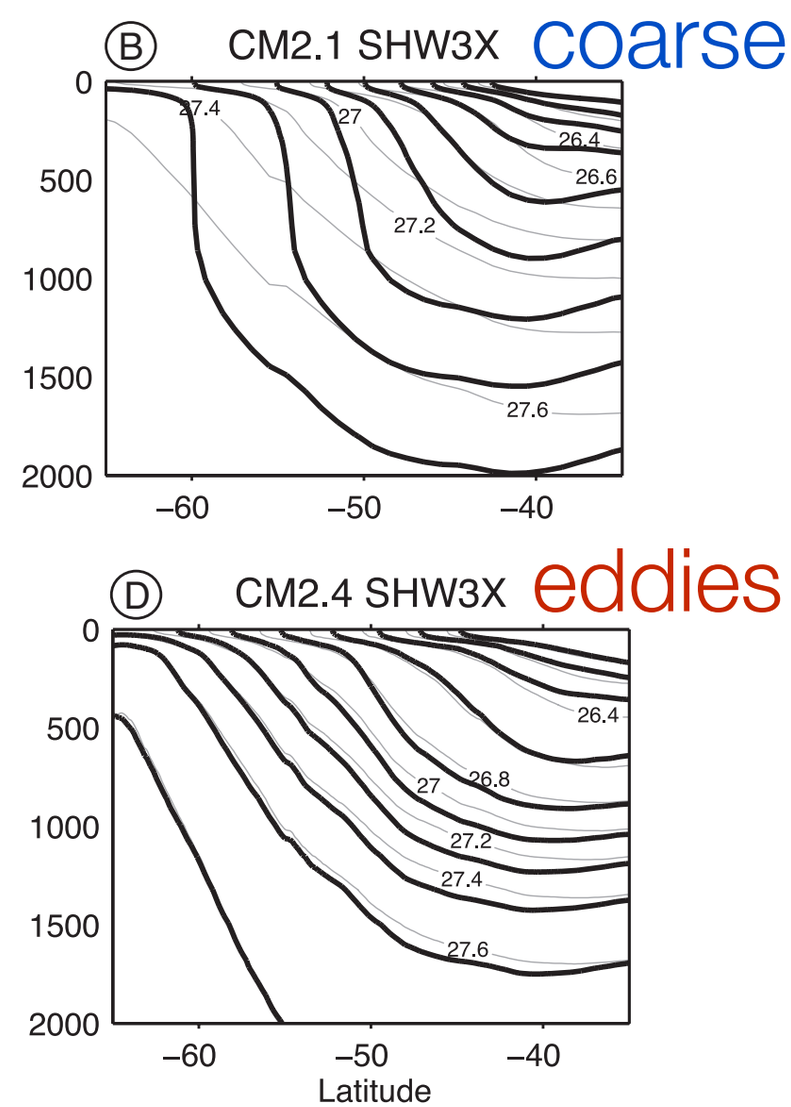
- COARSE resolution models show strong sensitivity to the winds

- EDDY RESOLVING / PERMITTING models are much less sensitive

  → eddies compensate for circulation changes due to winds

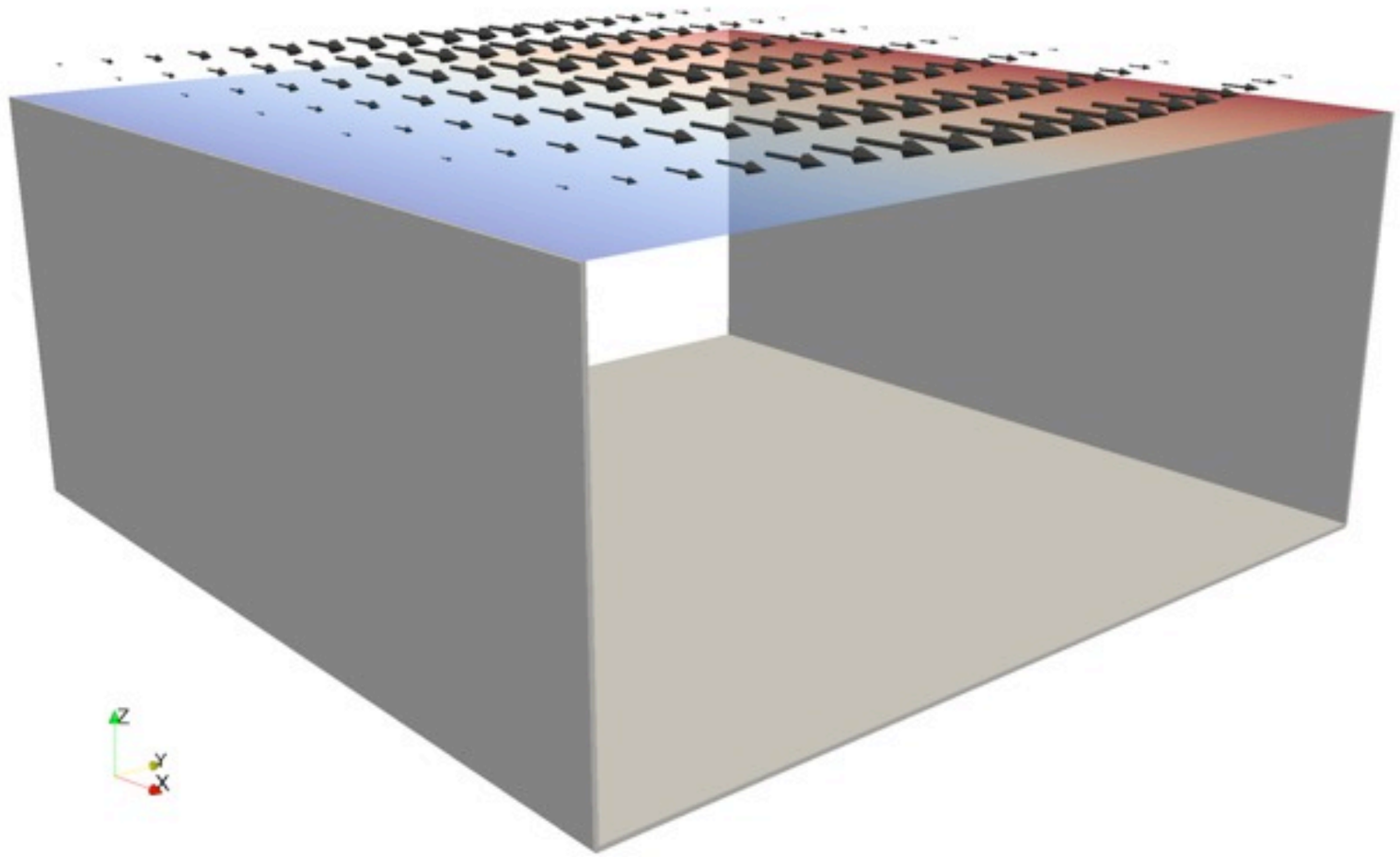
Found in everything from two-layer QG models up to coupled climate models

GFDL Model:
Triple Strength Winds



Farneti et al. 2010

Idealized Problem - Reentrant Channel



- Forced at surface with wind stress & relaxation to prescribed temperature
- Surface temperature is practically fixed, interior adjusts to wind forcing
- Very weak interior diapycnal mixing
- Very small surface buoyancy flux

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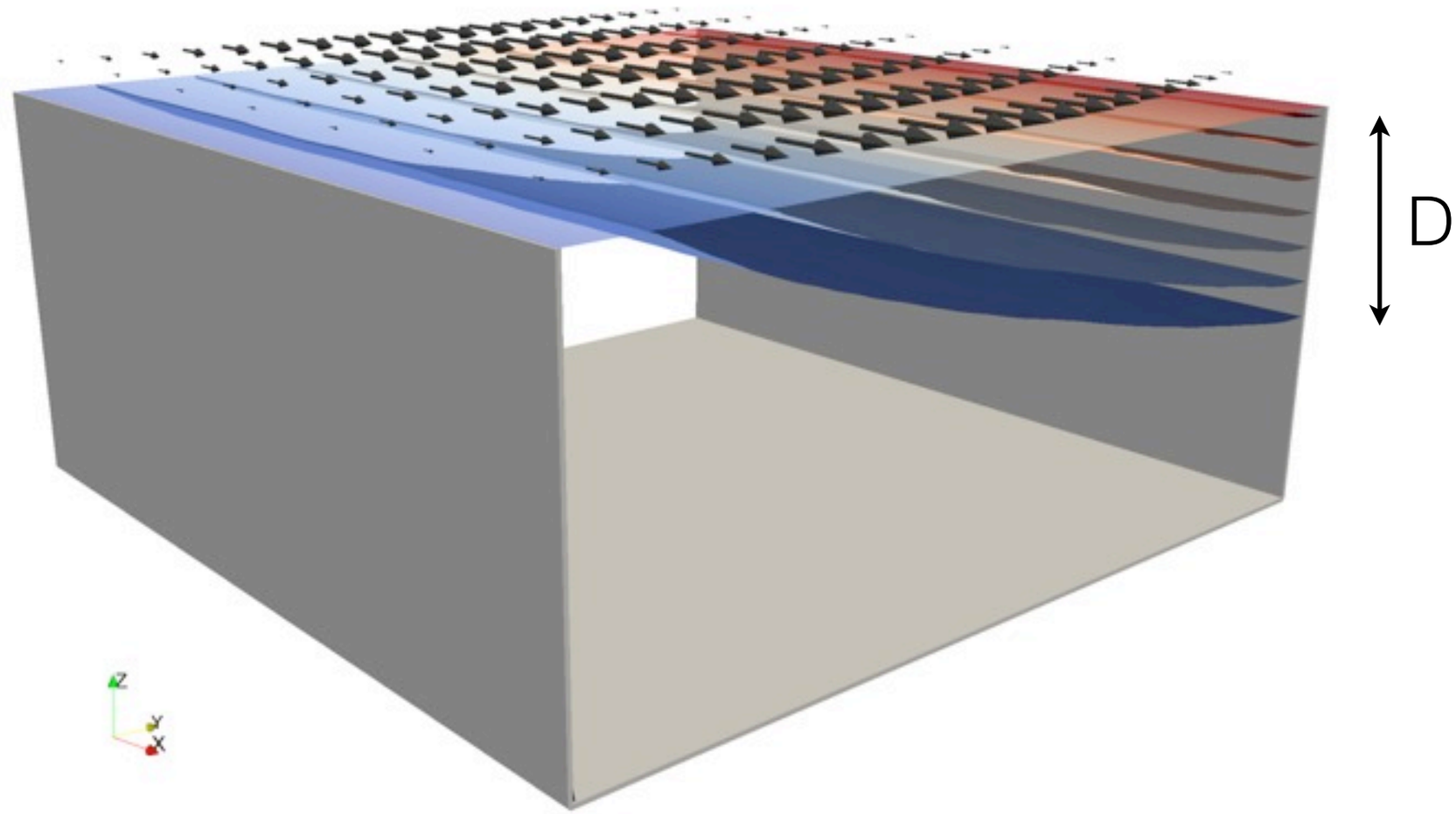

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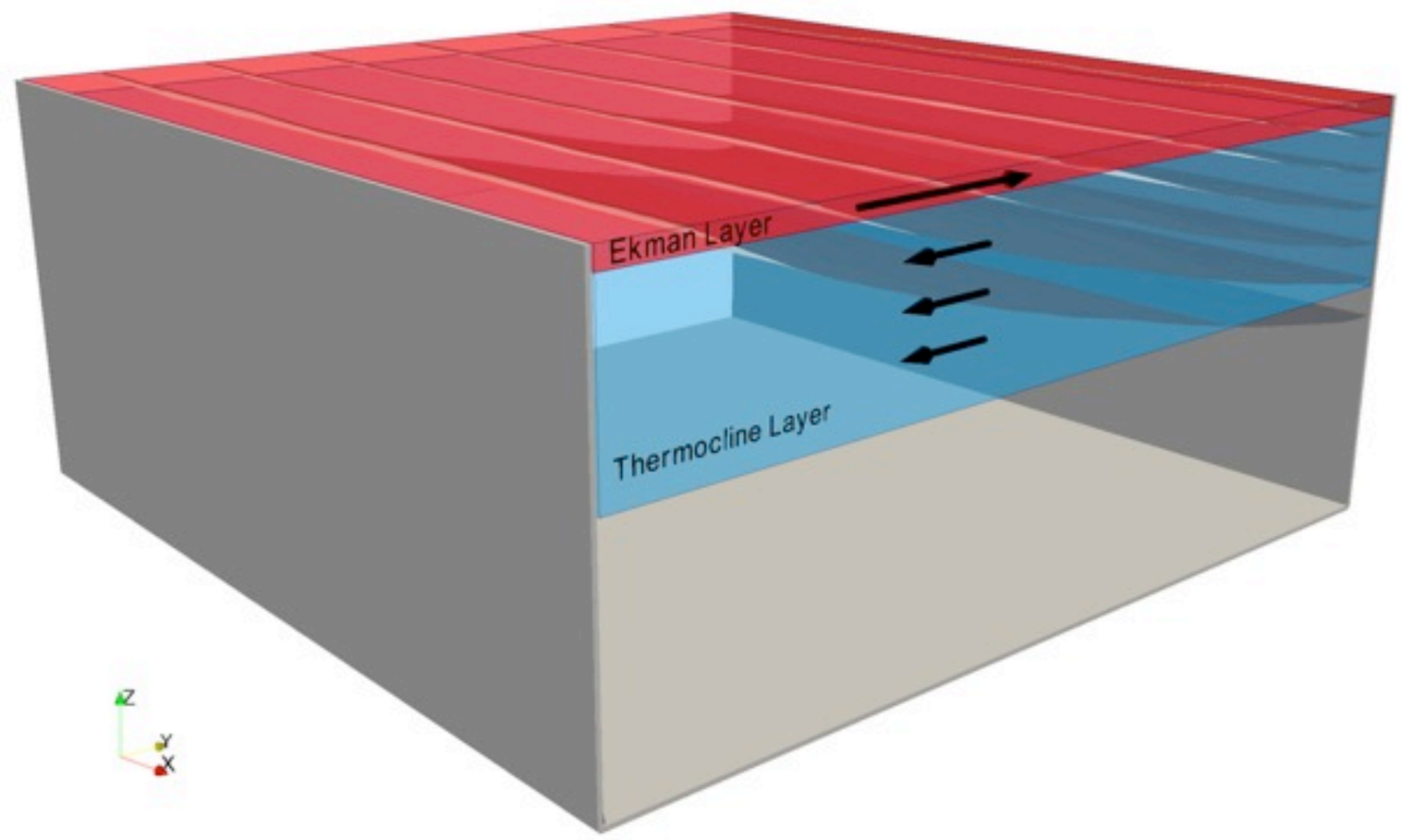
Equilibration of Circumpolar Currents: Meridional Heat Transport Perspective



$$\mathcal{H}_{ek} = -c_p f^{-1} \tau \theta_0$$

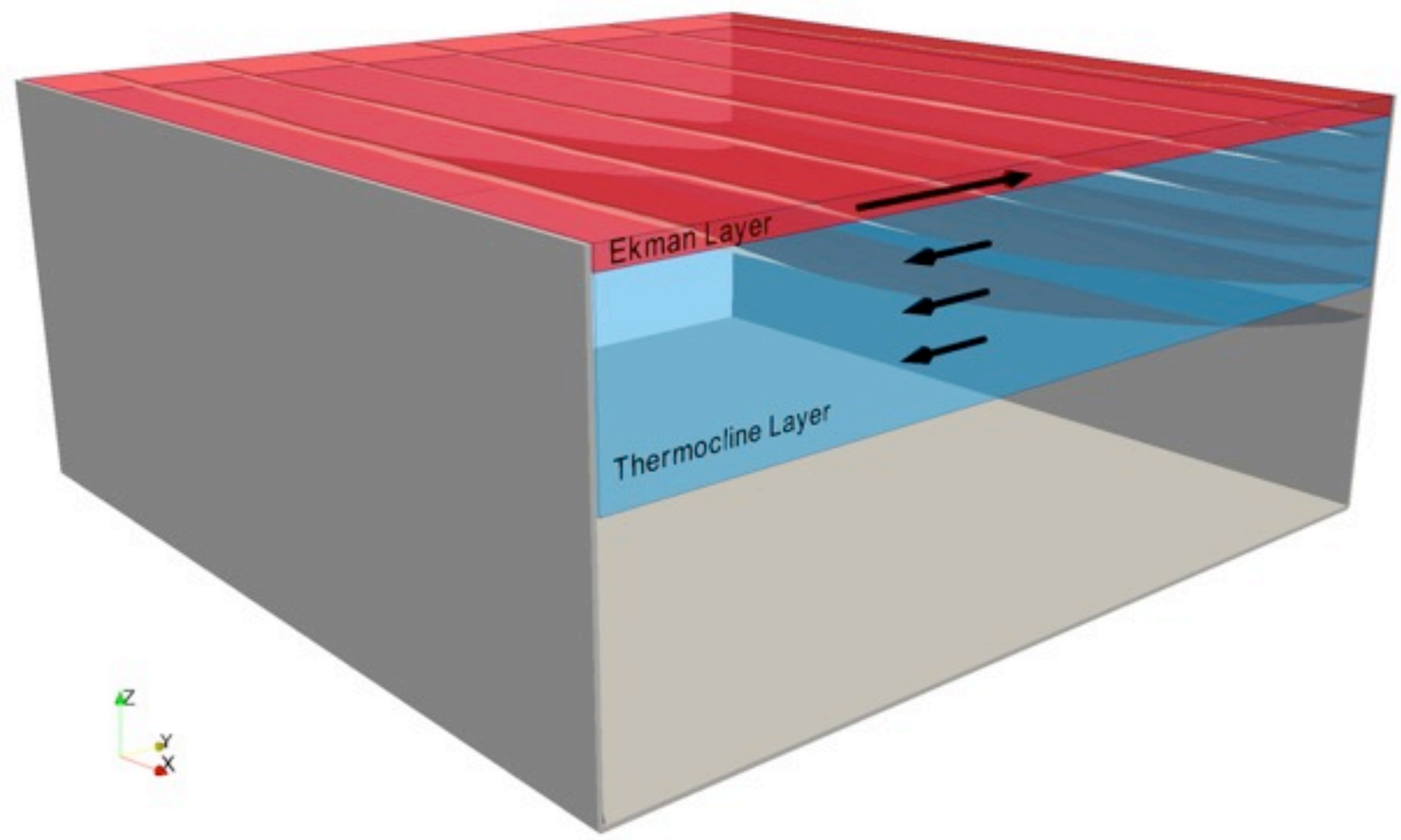
$$\mathcal{H}_g = \rho_0 c_p \int_{-D}^{0} \overline{v_g \theta} dz$$

$$= \rho_0 c_p D \widetilde{v_g \theta}$$

$$\boxed{H_{ek} + H_g \simeq 0}$$

$$D \simeq \frac{\tau \Delta\theta}{\rho_0 |f \widetilde{v_g \theta}|}$$

Equilibration of Circumpolar Currents: Meridional Heat Transport Perspective



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Gent-McWilliams Eddy Closure:

$$\widetilde{v_g \theta} \simeq -K_{GM} \Delta\theta / L_y$$

$$D \simeq \frac{\tau L_y}{\rho_0 |f| K_{GM}}$$

Equilibration of Circumpolar Currents: Meridional Heat Transport Perspective

"Macroturbulent" equilibration problem, similar to midlatitude atmosphere

Need a theory for K:
Green (1970), Stone (1972), Held & Larichev (1996), Schneider (2006), Jansen & Ferrari (2012)

Southern Ocean applications:
Johnson & Bryden (1989), Visbeck et al. (1997), Karsten et al. (2002), Marshall & Radko (2003), Olbers & Visbeck (2005), Cessi (2008), Abernathey et al. (2011)

Always based on zonal or "streamwise" average

$$\mathcal{H}_{ek} = -c_p f^{-1} \tau \theta_0$$

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Numerical Experiments

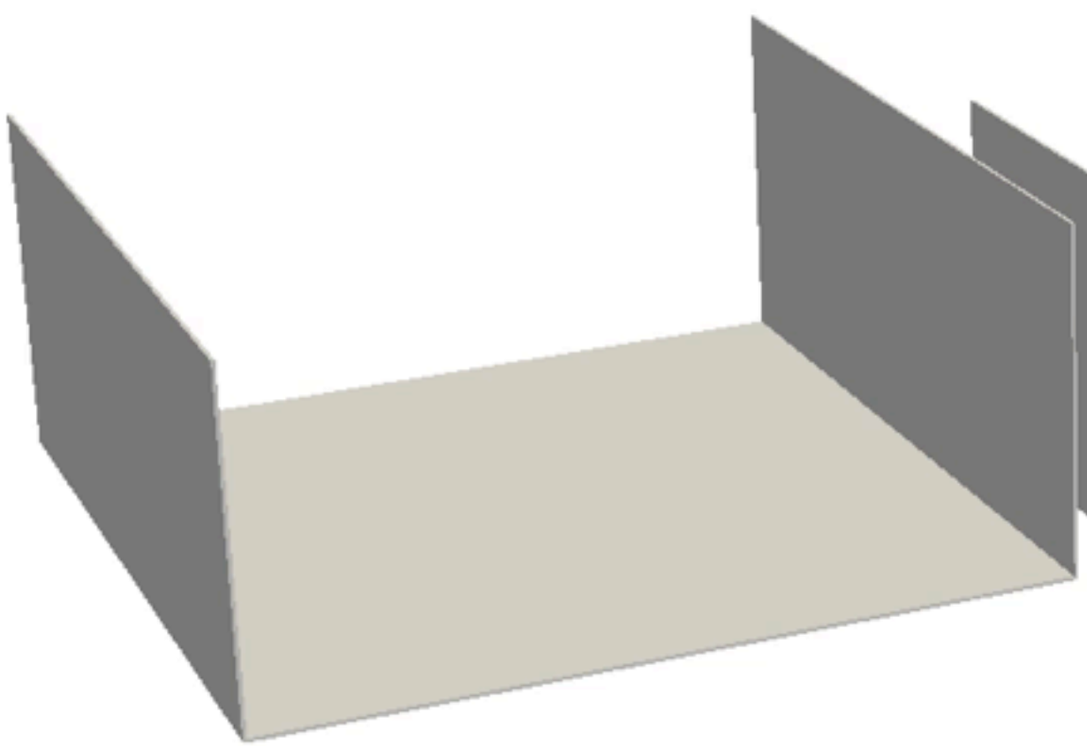
flat

ridge

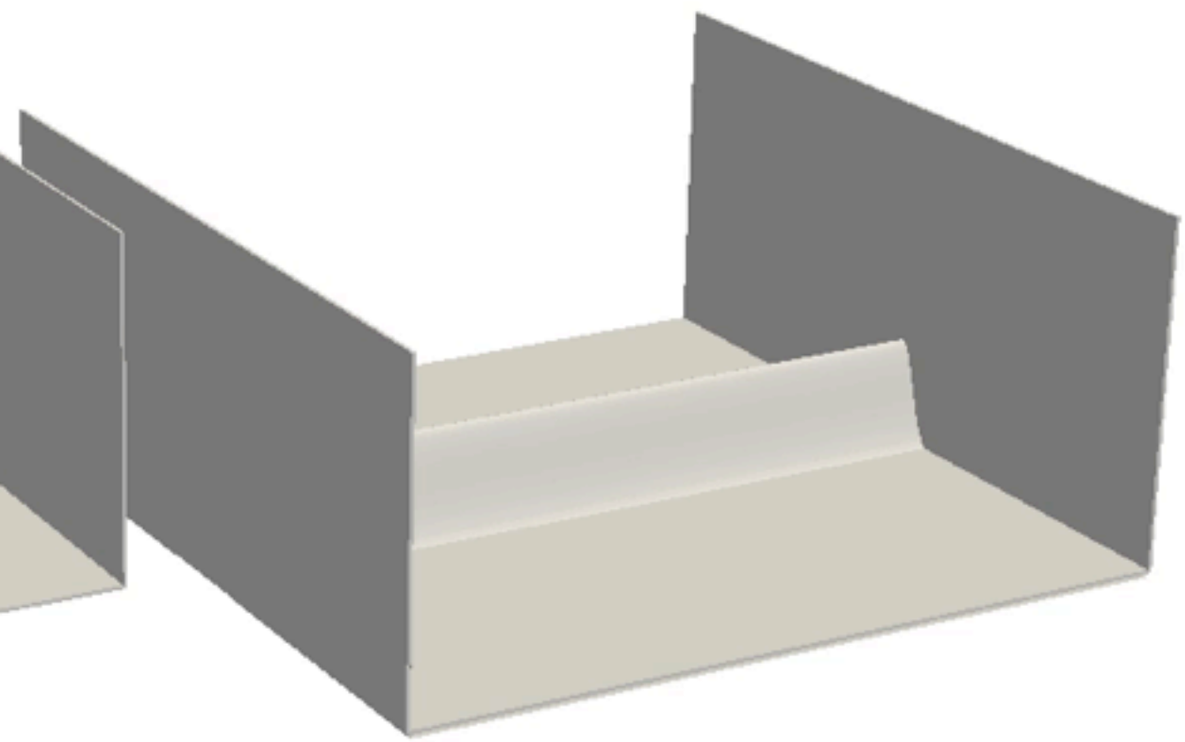
- MITgcm, 5km resolution, 40 vertical levels, adiabatic interior, temperature only
- 2000 km x 2000 km x 3000m, Gaussian ridge 1000 m high, 200 km wide
- Forced with sinusoidal wind jet, surface buoyancy restoring to linear gradient

Numerical Experiments

flat

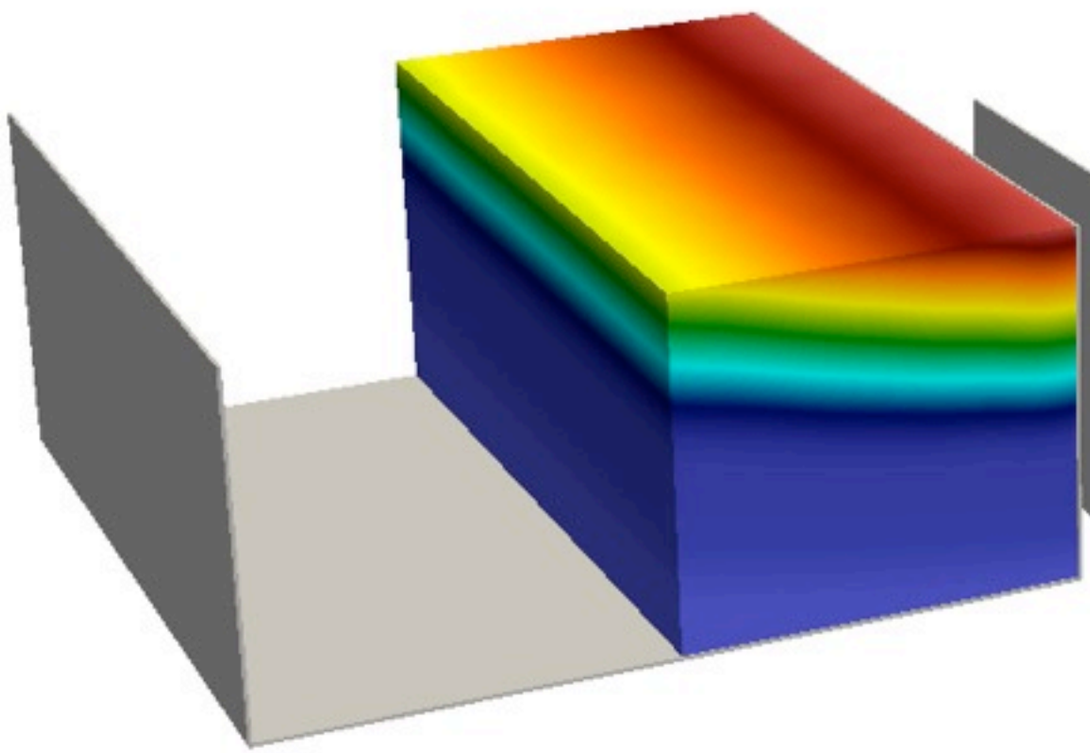


ridge

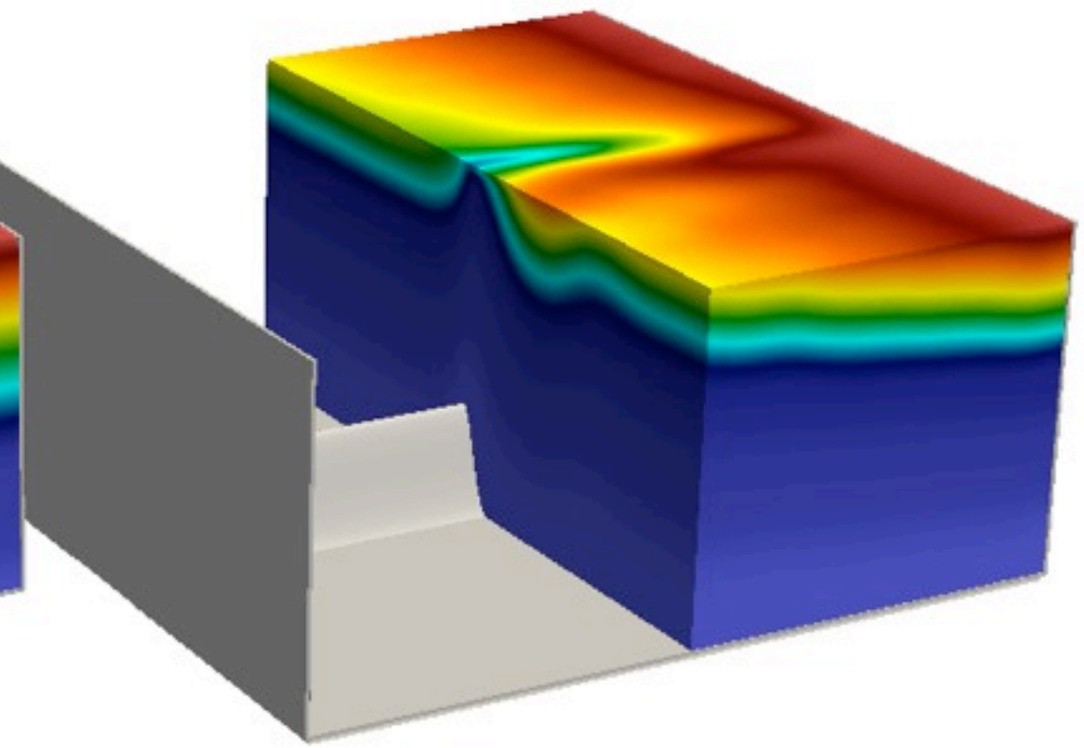


Numerical Experiments

flat



ridge



Numerical Experiments

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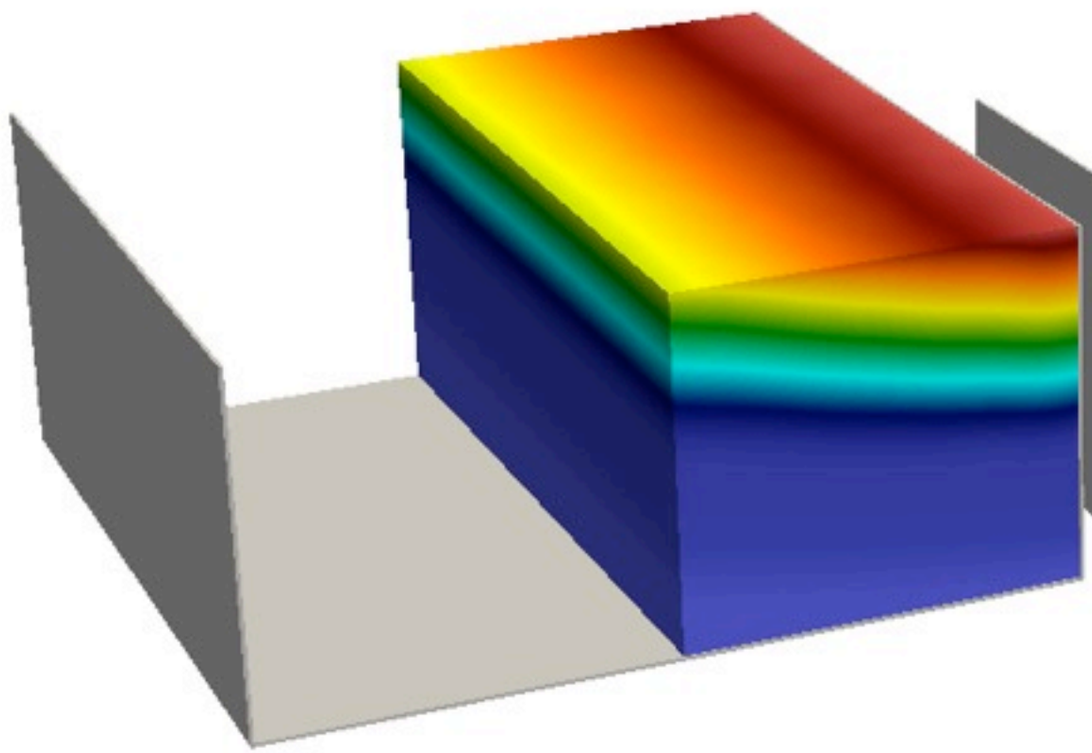
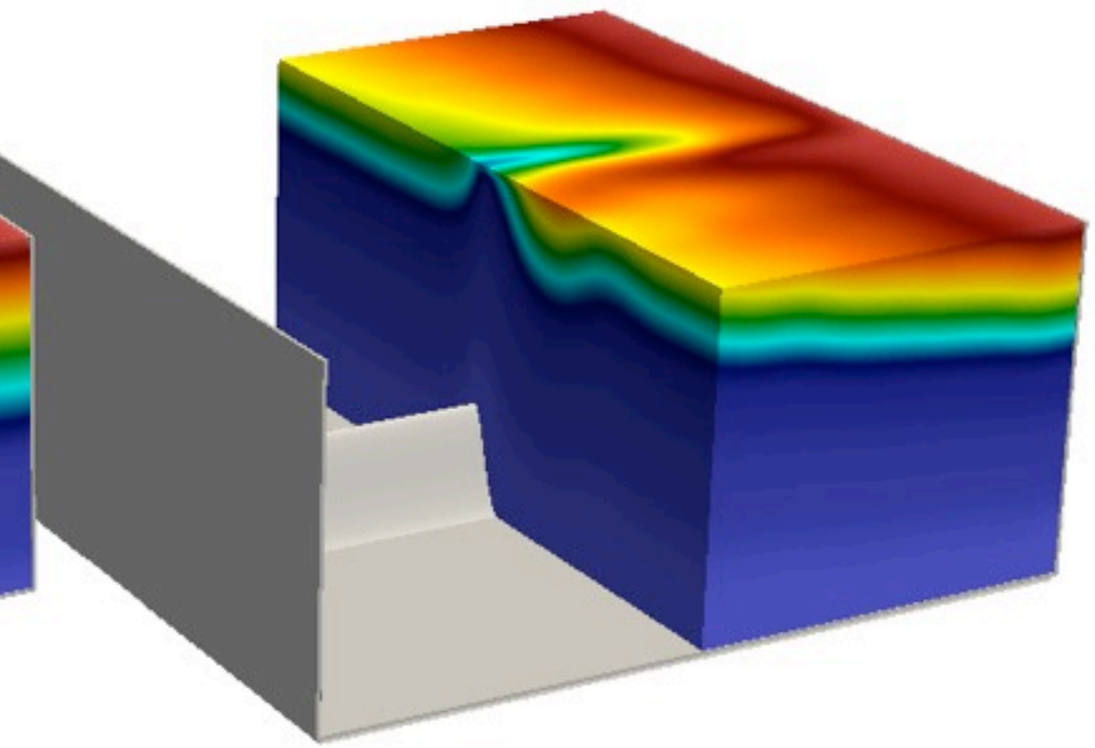


ridge



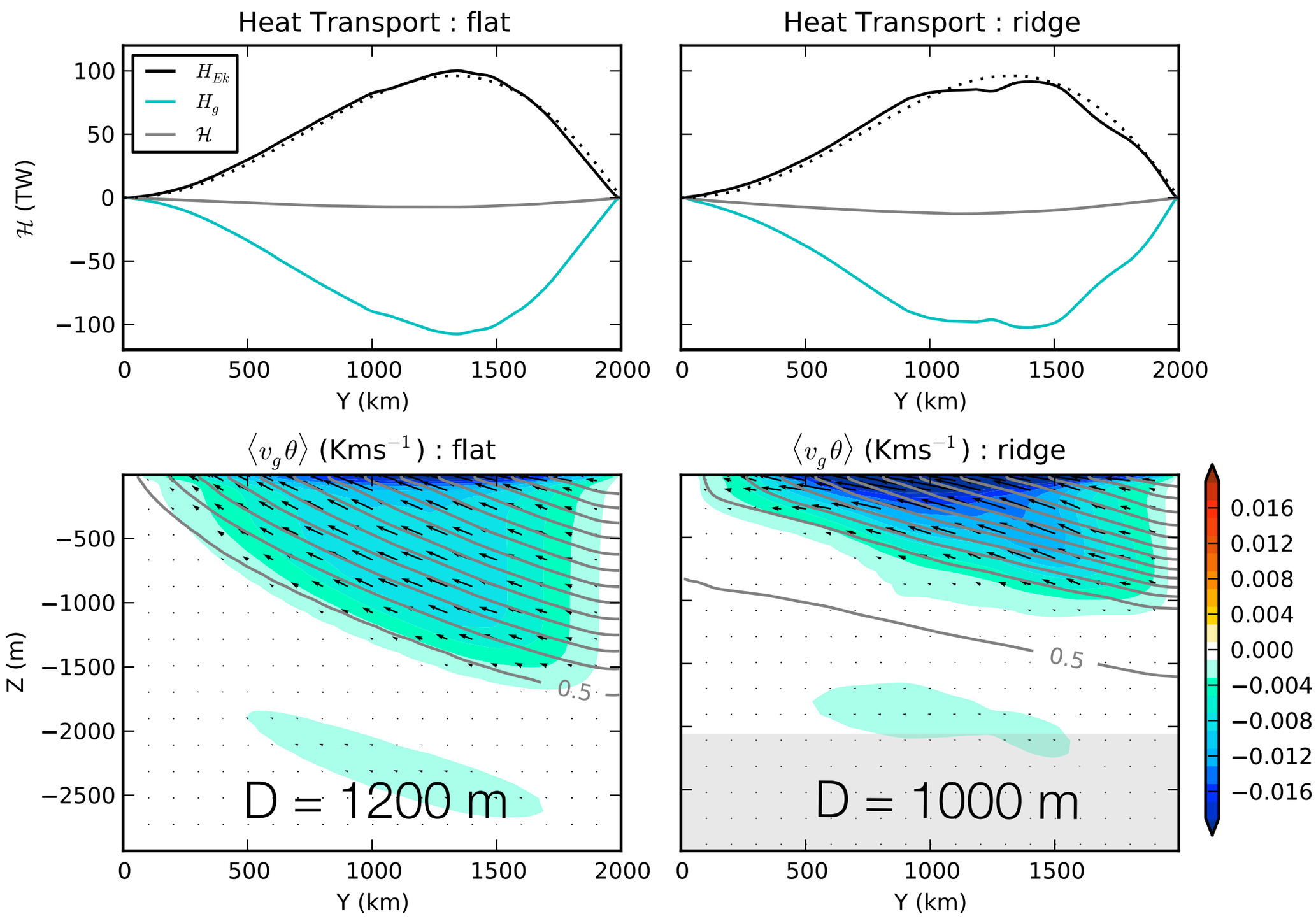
$$\langle \overline{v\theta} \rangle = \langle \overline{v} \rangle \langle \overline{\theta} \rangle + \langle \overline{v'\theta'} \rangle$$

$$\theta' = \theta - \overline{\theta}$$

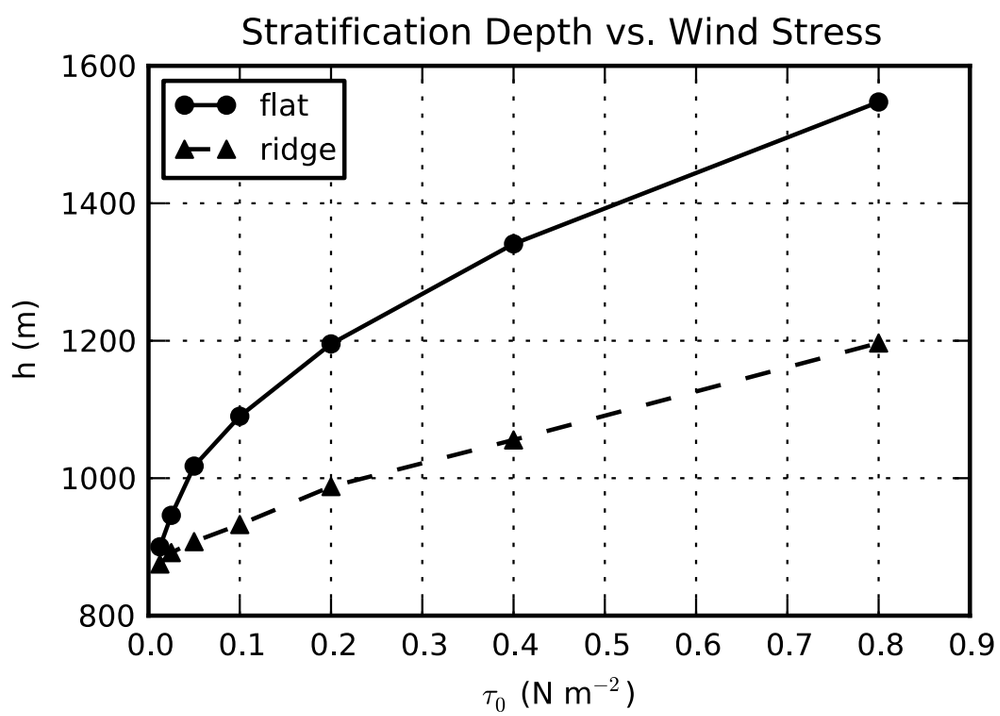
$$\langle \overline{v\theta} \rangle = \langle \overline{v} \rangle \langle \overline{\theta} \rangle + \langle \overline{v'\theta'} \rangle + \langle v^{\dagger}\theta^{\dagger} \rangle$$

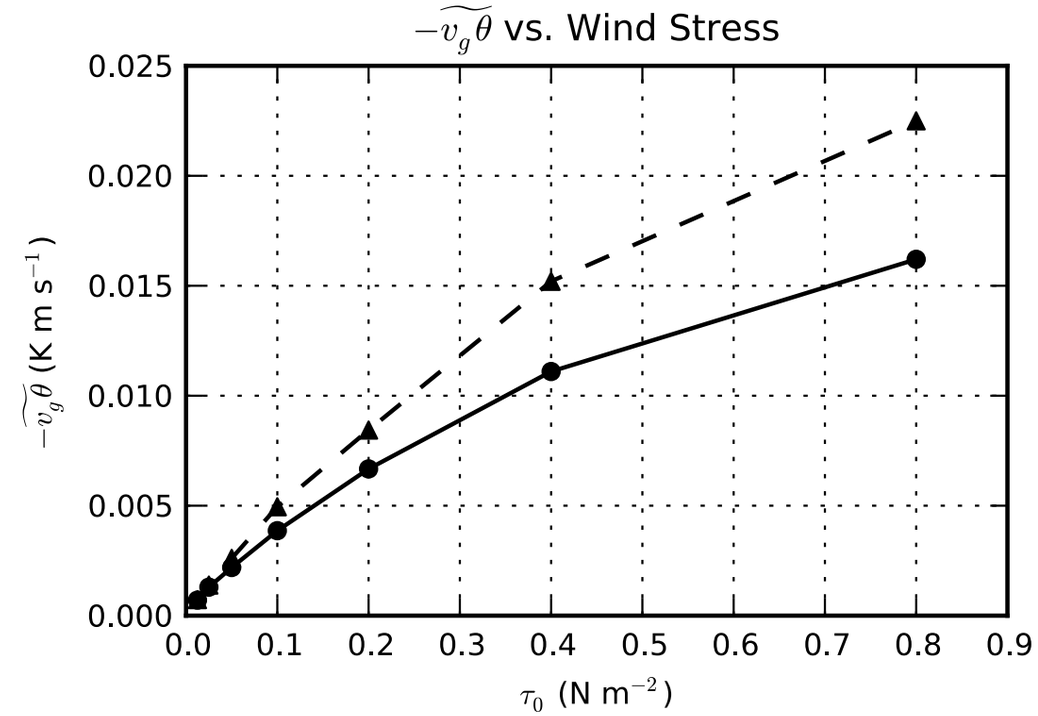
$$\theta^{\dagger} = \overline{\theta} - \langle \overline{\theta} \rangle$$

Stratification: with and without the ridge



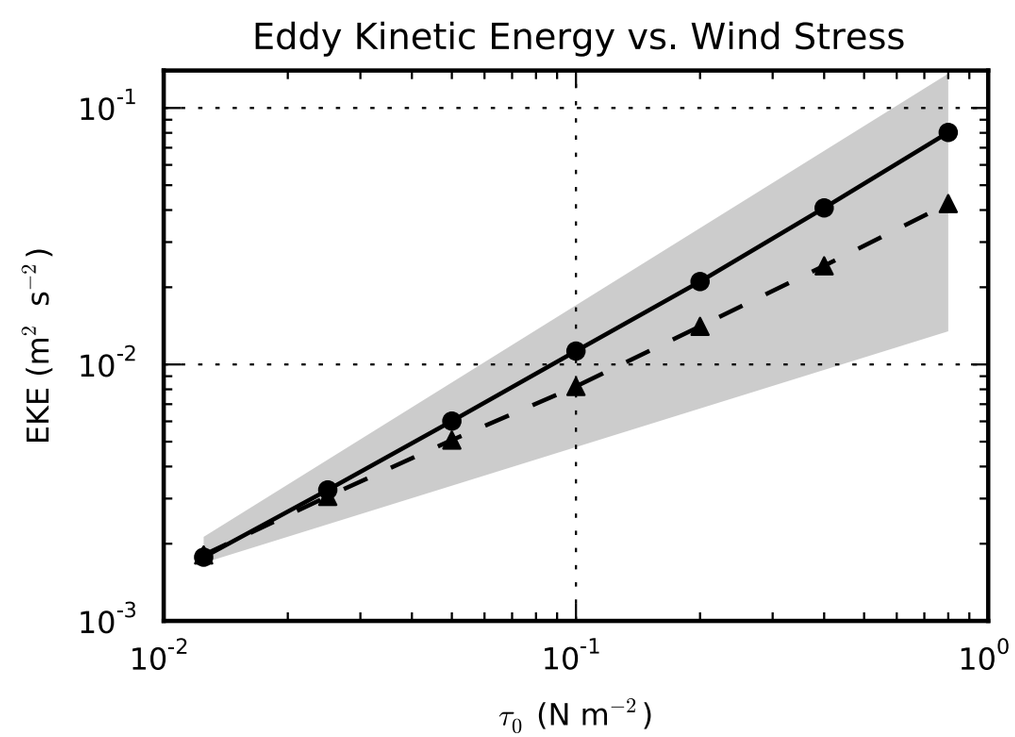
Sensitivity to Winds



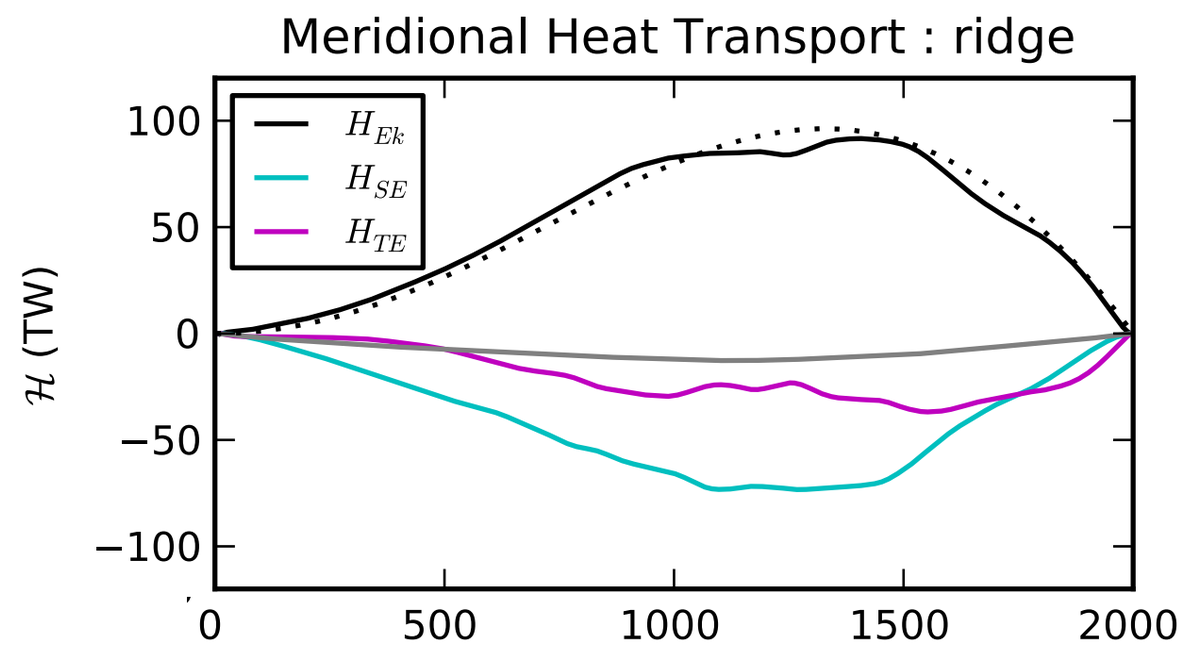


- With topography: shallower stratification and more intense geostrophic heat transport

- Even though EKE is lower!

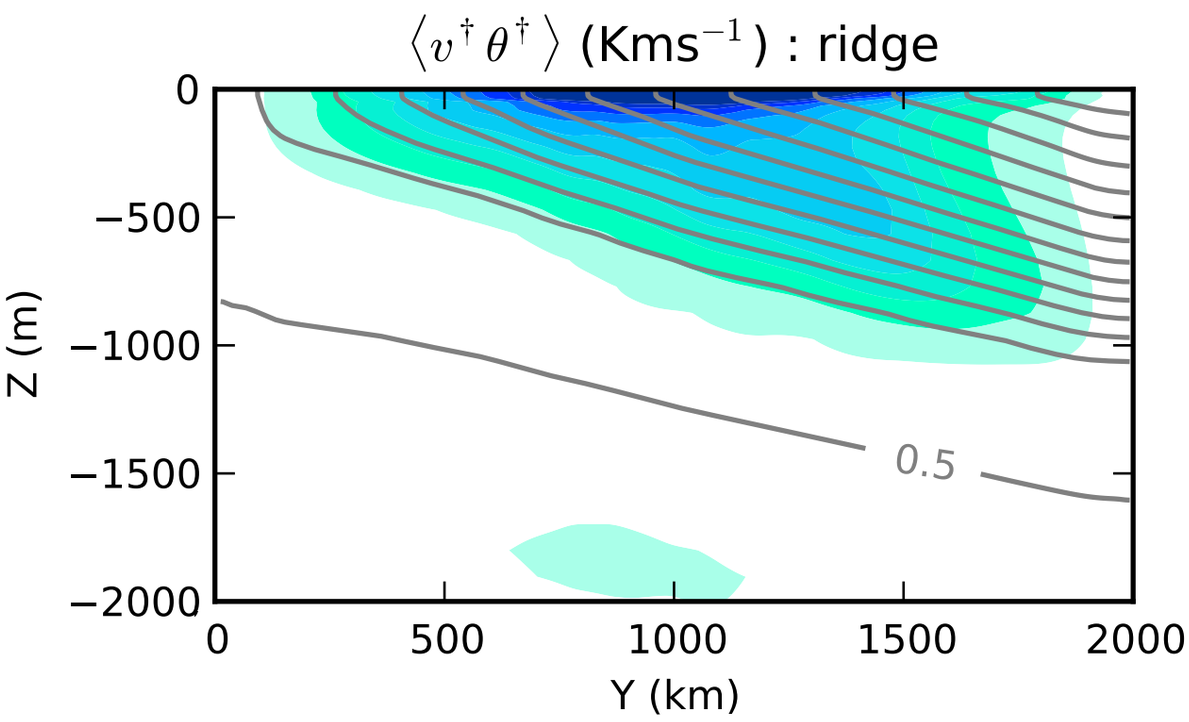


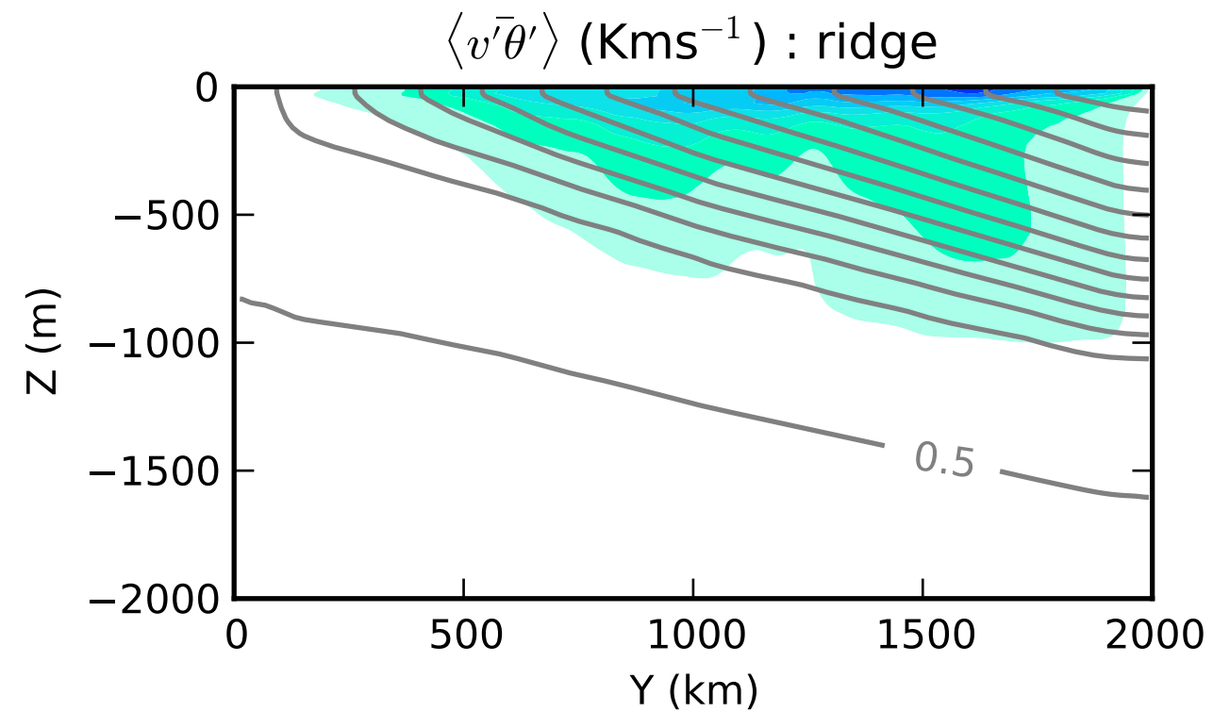
Standing vs. Transient Eddies

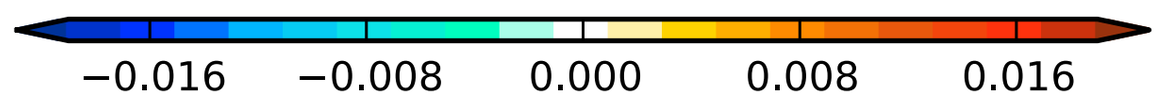


$$\mathcal{H}_g = \rho_0 c_p L_x \int_{-H}^{0} (\langle v^\dagger \theta^\dagger \rangle + \langle \overline{v'\theta'} \rangle) dz$$

$$\mathcal{H}_{SE} + \mathcal{H}_{TE}$$

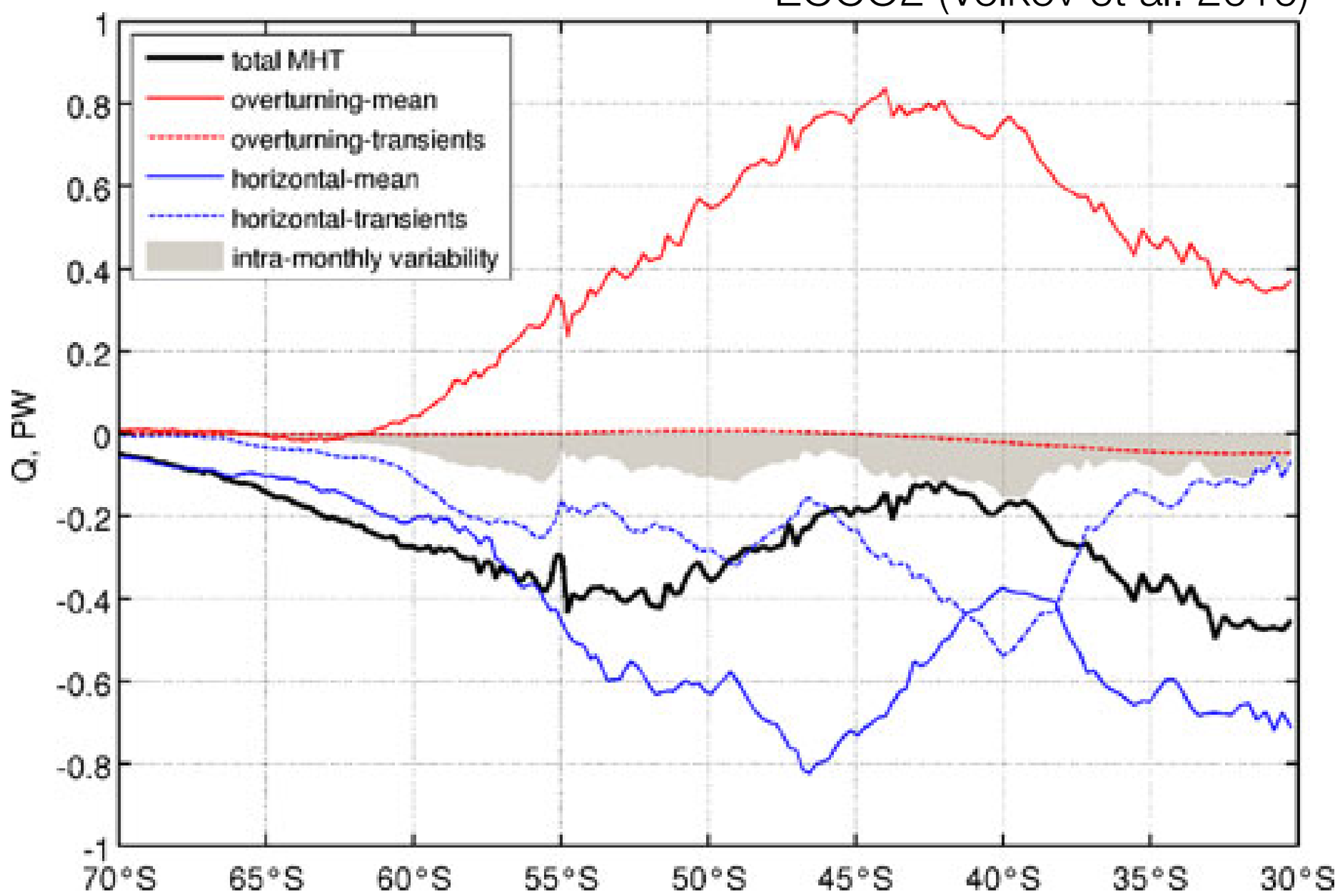






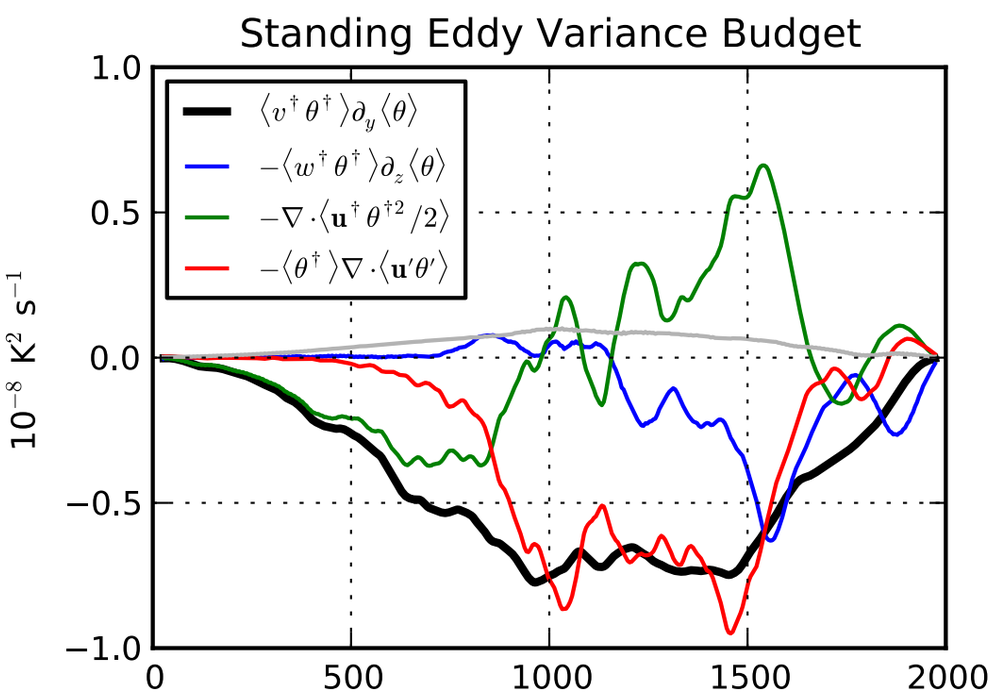
Meridional Heat Transport in “Realistic” Models

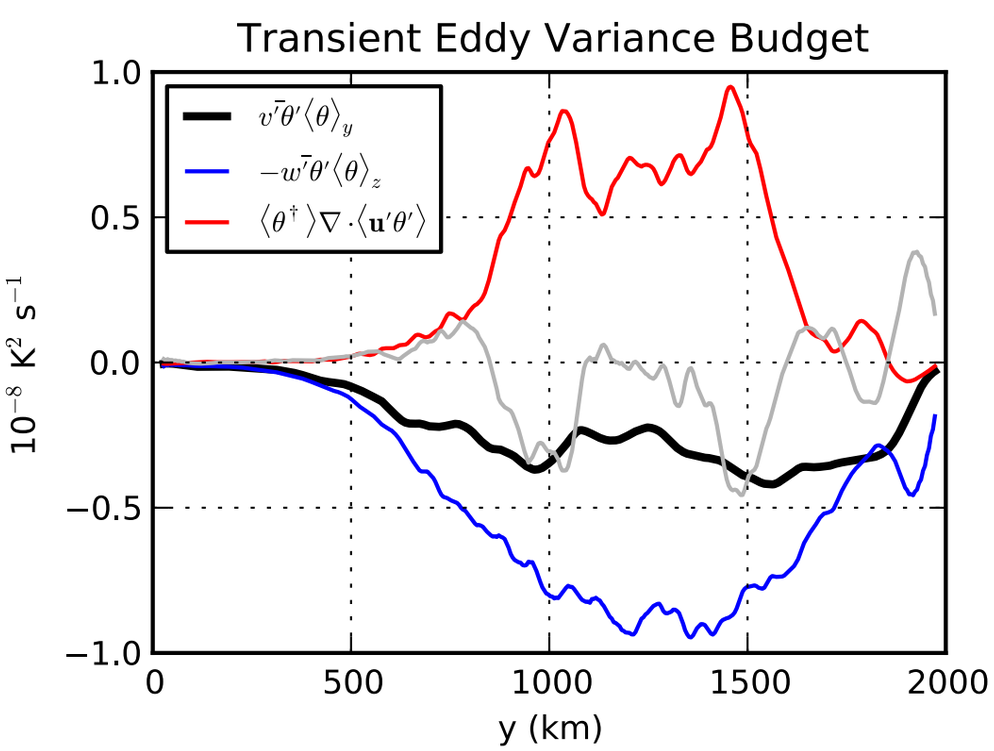
ECCO2 (Volkov et al. 2010)



- Qualitatively similar to simple model: standing eddies are the main term balancing the Ekman layer heat transport

Why do Standing Eddies Transport Heat? Variance Budget for $\theta^\dagger$





standing variance

$$\langle v^\dagger \theta^\dagger \rangle \langle \theta \rangle_y \simeq -\langle w^\dagger \theta^\dagger \rangle \langle \theta \rangle_z - \nabla \cdot \frac{\langle \boldsymbol{u}^\dagger \theta^{\dagger 2} \rangle}{2} - \langle \theta^\dagger \nabla \cdot (\overline{\boldsymbol{u}'\theta'}) \rangle$$

conversion to transient variance

transient variance

$$\langle \overline{v'\theta'} \rangle \langle \theta \rangle_y \simeq -\langle \overline{w'\theta'} \rangle \langle \theta \rangle_z + \langle \theta^\dagger \nabla \cdot (\overline{\boldsymbol{u}'\theta'}) \rangle$$

large

EKE → Dissipation

Two-Layer QG Model

qgpv:

$$q_1 = \nabla^2\psi_1 + \beta y + F_1(\psi_2 - \psi_1)$$

$$q_2 = \nabla^2\psi_2 + \beta y + F_2(\psi_1 - \psi_2) + \frac{f_0 h_b}{H_2}$$

zonal mean, standing, transient:

$$\psi_{1,2} = <\psi_{1,2}>(y) + \psi^{\dagger}_{1,2}(x,y) + \psi'_{1,2}(x,y,t)$$

$$q_{1,2} = <q_{1,2}>(y) + q^{\dagger}_{1,2}(x,y) + q_{1,2}(x,y,t)\,,$$

transient eddy closure: $\overline{J(\psi'_{1,2}, q'_{1,2})} = -K\nabla^2\overline{q}_{1,2}$

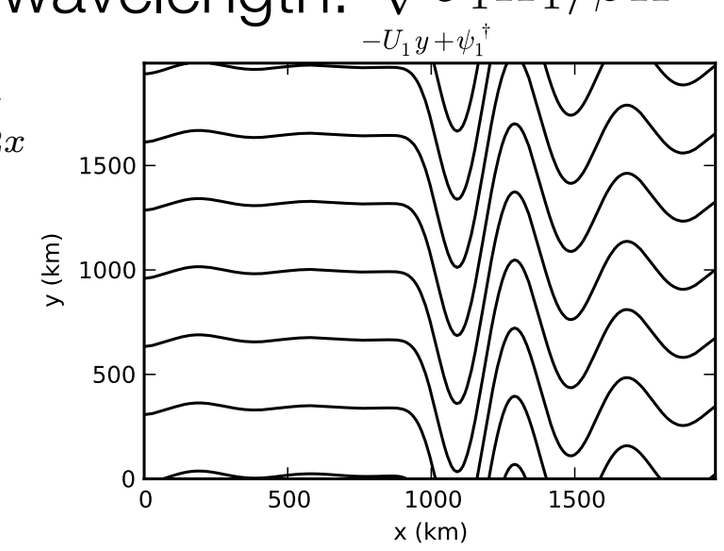
$$\begin{aligned} U_1\psi^{\dagger}_{1xx} + (\beta - F_1U_2)\psi^{\dagger}_1 + F_1U_1\psi^{\dagger}_2 &= Kq^{\dagger}_{1x} \\ U_2\psi^{\dagger}_{2xx} + (\beta - F_2U_1)\psi^{\dagger}_2 + F_2U_2\psi^{\dagger}_1 + U_2 f_0\frac{h_b}{H_2} &= Kq^{\dagger}_{2x} - r\psi^{\dagger}_{2x} \end{aligned}$$

large scale approximation: topography scale larger than deformation radius

1st order, equivalent barotropic: $U_2\psi^{\dagger}_1 = U_1\psi^{\dagger}_2$

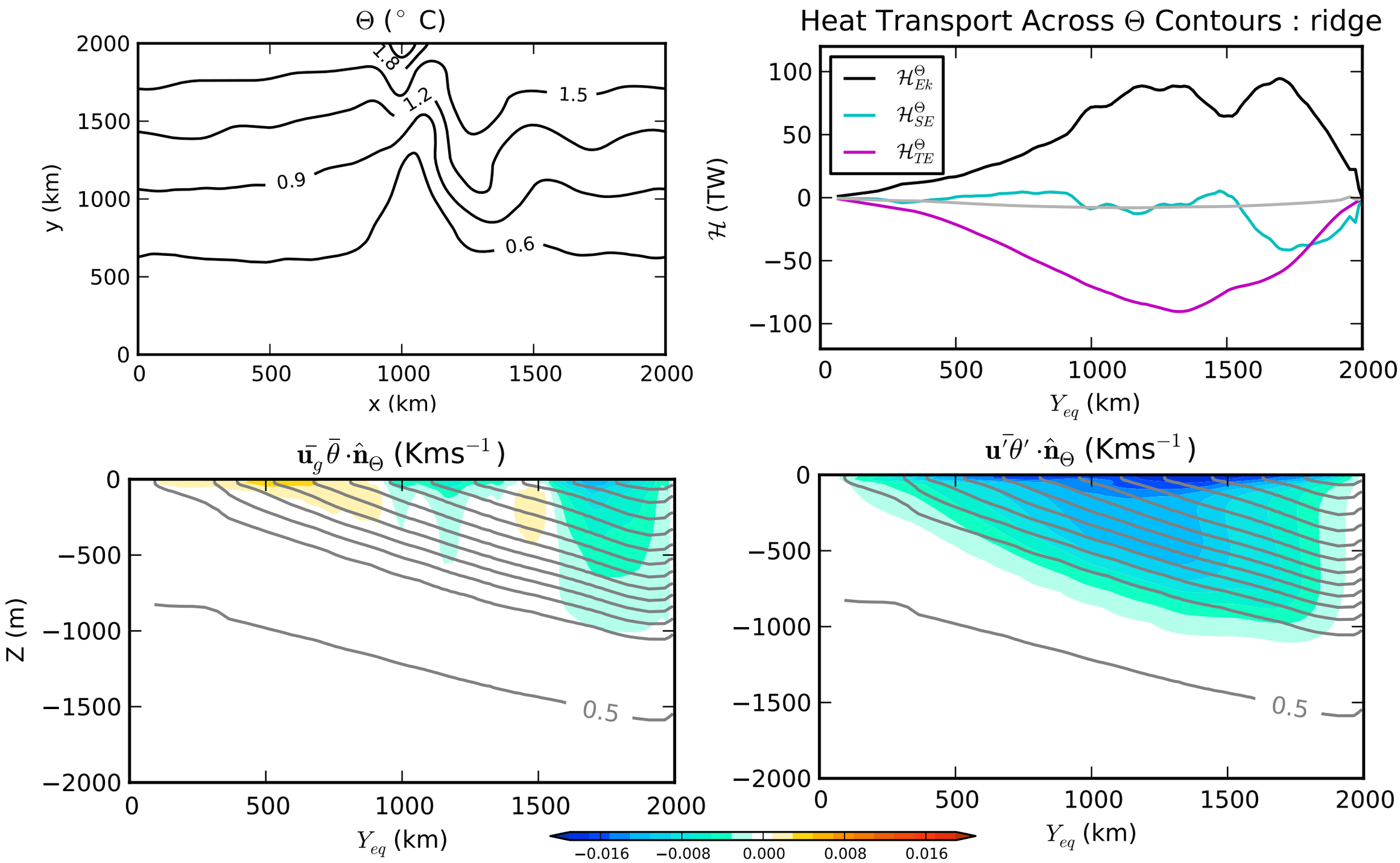
wavelength: $\sqrt{U_1H_1/\beta H}$

next order: $\left(1 + \frac{H_1U_1^2}{H_2U_2^2}\right)\psi^{\dagger}_{2xx} + \frac{\beta}{U_2}\left(1 + \frac{H_1U_1}{H_2U_2}\right)\psi^{\dagger}_2 + f_0\frac{h_b}{H_2} = -\frac{r}{U_2}\psi^{\dagger}_{2x}$



upper layer PV flux (prop. to heat flux) $KF_1(U_1 - U_2)\left(1 + \frac{<\psi^{\dagger 2}_{1x}>}{U_1^2}\right) = \frac{\tau}{\rho_0 H_1}$

Cross-Stream Heat Transport



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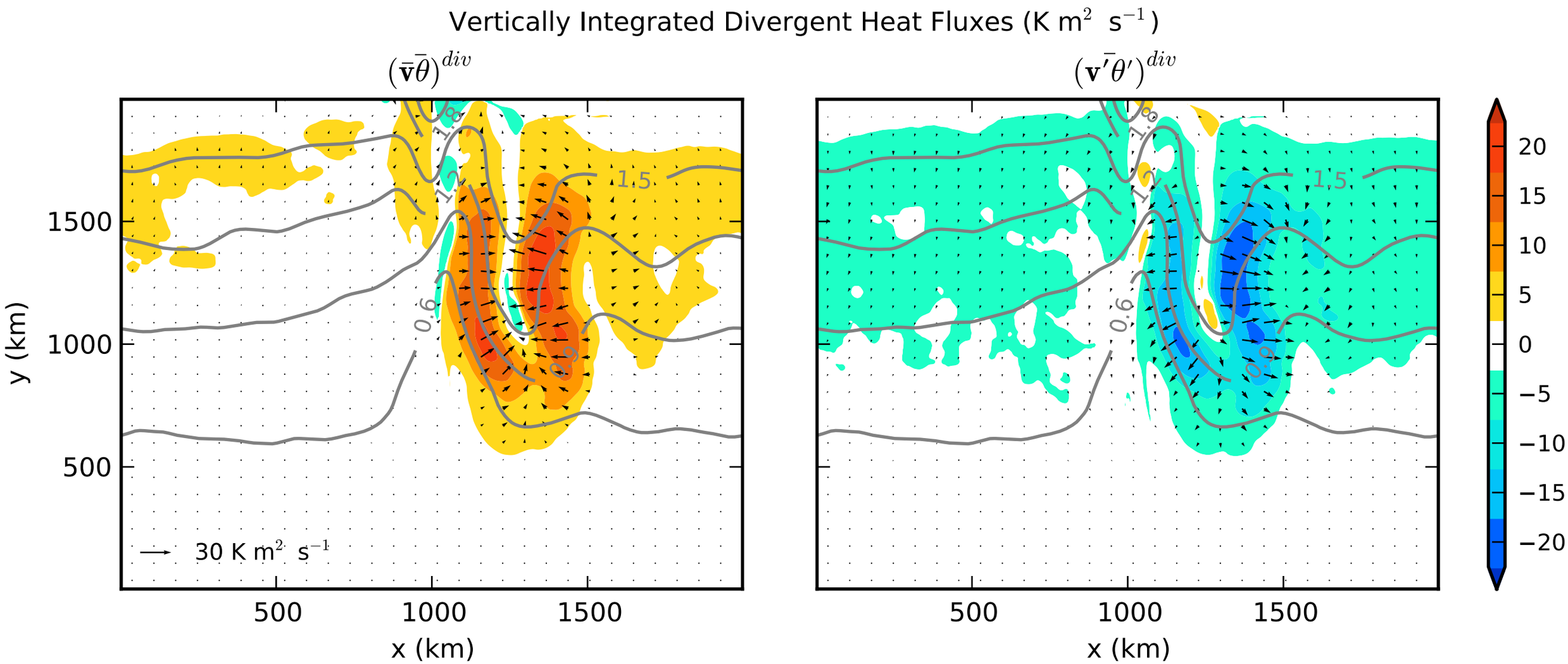
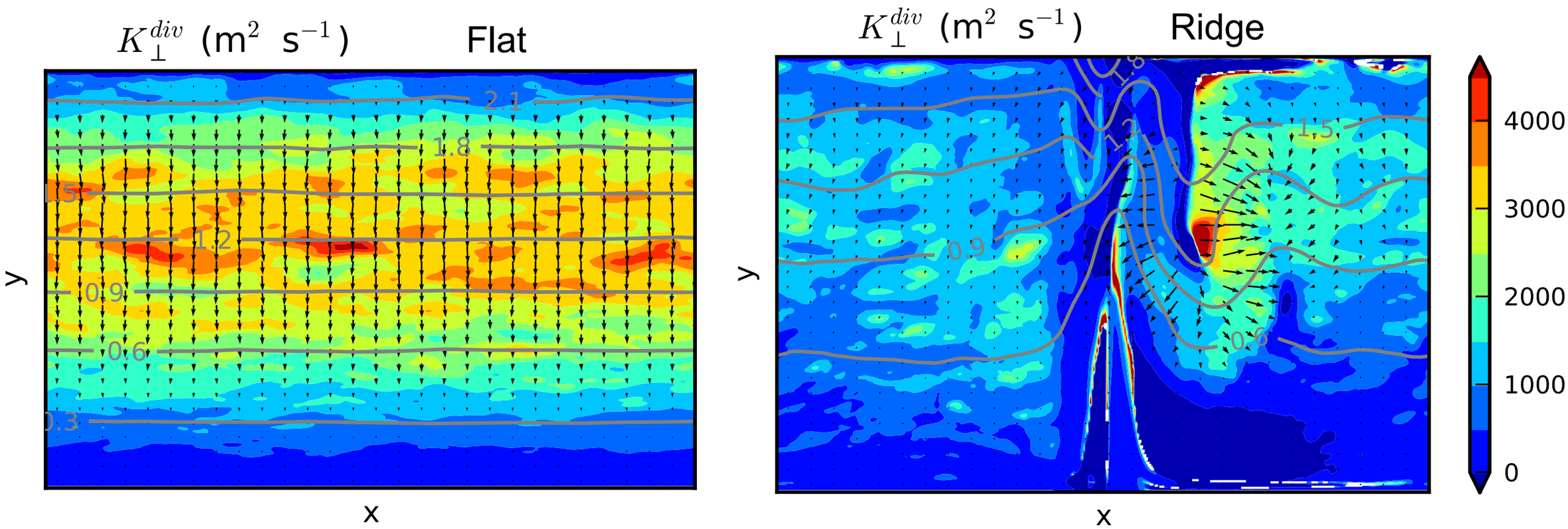


Transient Eddy Diffusivity

$$K_{\perp}^{div}(x,y) = -\frac{1}{H}\frac{\boldsymbol{F}_{TE}^{div}\cdot\hat{\boldsymbol{n}}}{|\nabla\Theta|}$$



$$\mathcal{H}_{TE}^{\Theta} = \rho_0 c_p \oint_{\Theta} \boldsymbol{F}_{TE}^{div}\cdot\hat{\boldsymbol{n}}ds = -\rho_0 c_p \oint_{\Theta} K_{\perp}^{div}|\nabla\Theta|ds$$

Can K be removed from the integral?

Two Paradigms of Baroclinic Instability

Axisymmetric Background State

- Charney (1947); Eady (1949); Phillips (1951); Gill, Green & Simmons (1974)
- Growth rates depend on a single profile of shear and N^2
- Modes can propagate in space
- Barotropic mean flow has no affect on stability

Zonally Varying Background State

- Absolute / local vs. convective / global instability: Merkine (1977); Pierrehumbert (1984)
- Locally unstable modes depend on mean shear *and* its zonal gradient
- Modes are locked in place
- Barotropic mean flow has a strong effect on stability

Conclusions

- The depth of the stratification depends on the efficiency of eddies (transient or standing) at transporting heat southward: less efficient eddies require a shallower thermocline

- ZONAL average view: when a topographic ridge is present, the standing eddies appear to do most of the work of restratifying the channel; “eddy saturation” is between Ekman flux and standing eddies

- STREAMWISE average view: the standing component is greatly reduced, and the transients are dominant; just like the case with no topography, “eddy saturation” is between Ekman flux and transient eddies.

- BUT, unlike the flat-bottom case, in the ridge experiments the relevant transient eddy fluxes are highly localized downstream of the topography; strong along-stream anisotropy complicates closure attempt

de Szoeke & Levine

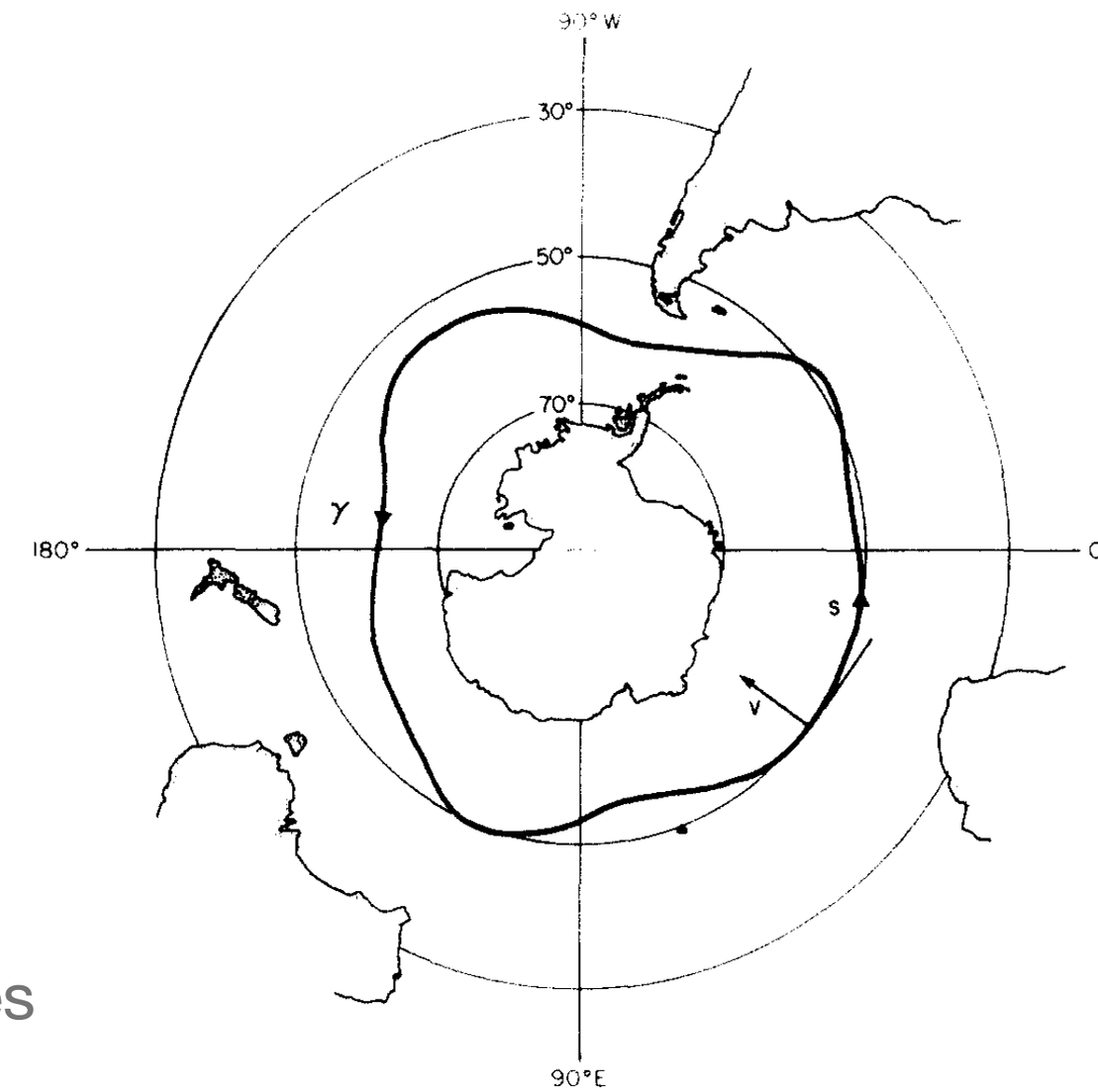
- Computed heat transport across an ACC contour (used depth-averaged temperature) from observations

- Considered three components:

$$F_{\mathrm{Ek}} = -\int_{\gamma} c f^{-1}(\theta_{sfc} - \theta_0)\tau^{(s)}\,\mathrm{d}s,$$

$$F_{\mathrm{BC}} = \int_{\gamma} \mathrm{d}F_{\mathrm{BC}}(s),$$

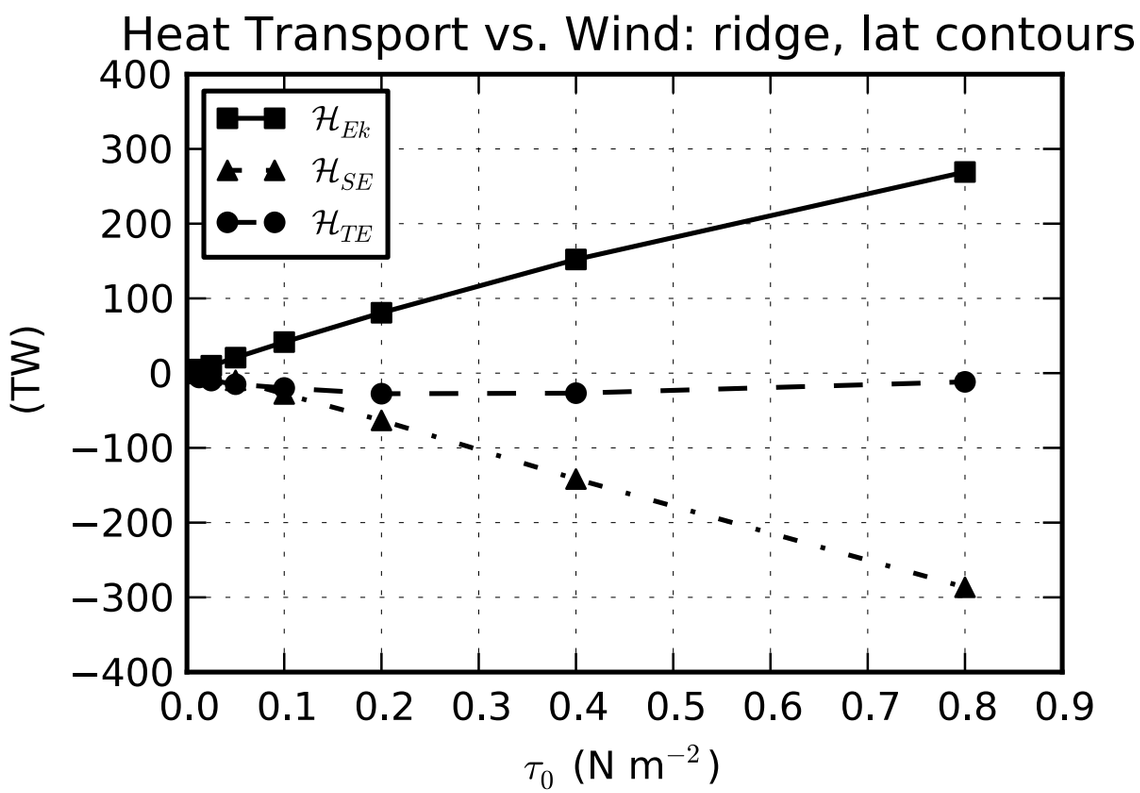
$$F_{\mathrm{BT}} = \iint_{\gamma} c\rho v_{g0}(\bar{\theta} - \theta_0)\,\mathrm{d}A,$$

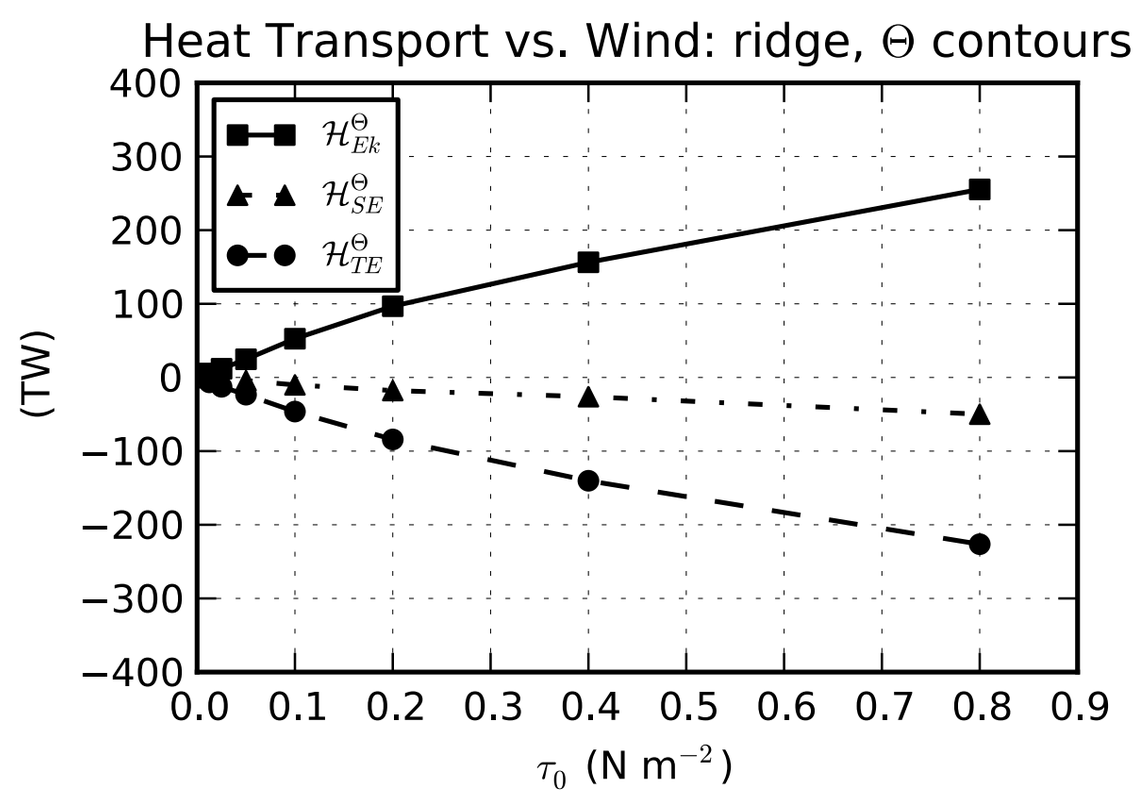


- Found that the geostrophic mean flow carries almost no heat

	F_{BC}	F_{BT}	F_{Ek}	F_{E}	$F_?$	Q
estimate:	0	0	−15	45	0	+30
error:	±23	±10		±30?		
units:	10^{13} W					

Meridional Heat Transport: Dependence on Winds





Why do Standing Eddies Transport Heat? Variance Budget for $\theta^\dagger$

(1) temperature equation:

$$\theta_t + u \cdot \nabla\theta = \kappa_h \nabla_h^2 \theta + (\kappa_v \theta_z)_z - \frac{\lambda}{\delta}(\theta_s - \theta^*)$$

(2) zonal / time average temperature equation:

$$\langle\theta\rangle_t + \langle u\rangle \cdot \nabla\langle\theta\rangle + \nabla \cdot \langle u^\dagger \theta^\dagger\rangle + \nabla \cdot \langle\overline{u'\theta'}\rangle = \kappa_h \nabla_h^2 \langle\theta\rangle + (\langle\kappa_v \theta_z\rangle)_z - \frac{\lambda}{\delta}(\langle\theta\rangle_0 - \theta^*)$$

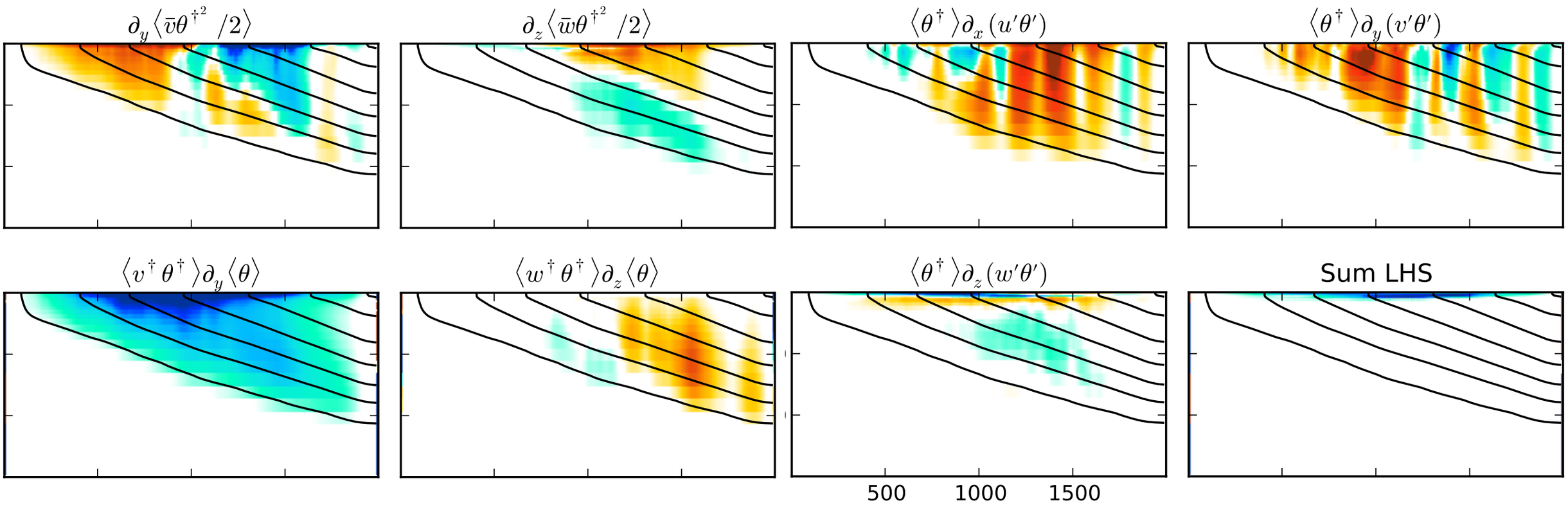
Subtract (2) from (1), multiply by $\theta^\dagger$, take another zonal / time average

(3) standing wave variance equation

$$\nabla \cdot \left(\langle u\rangle \frac{\langle\theta^{\dagger 2}\rangle}{2} + \frac{\langle u^\dagger \theta^{\dagger 2}\rangle}{2} \right) + \langle u^\dagger \theta^\dagger\rangle \cdot \nabla\langle\theta\rangle = -\langle\theta^\dagger \nabla \cdot (\overline{u'\theta'})\rangle - \kappa_h \langle|\nabla\theta^\dagger|^2\rangle - \langle\kappa_v |\theta_z^\dagger|^2\rangle - \frac{\lambda}{\delta}\langle\theta_s^{\dagger 2}\rangle$$

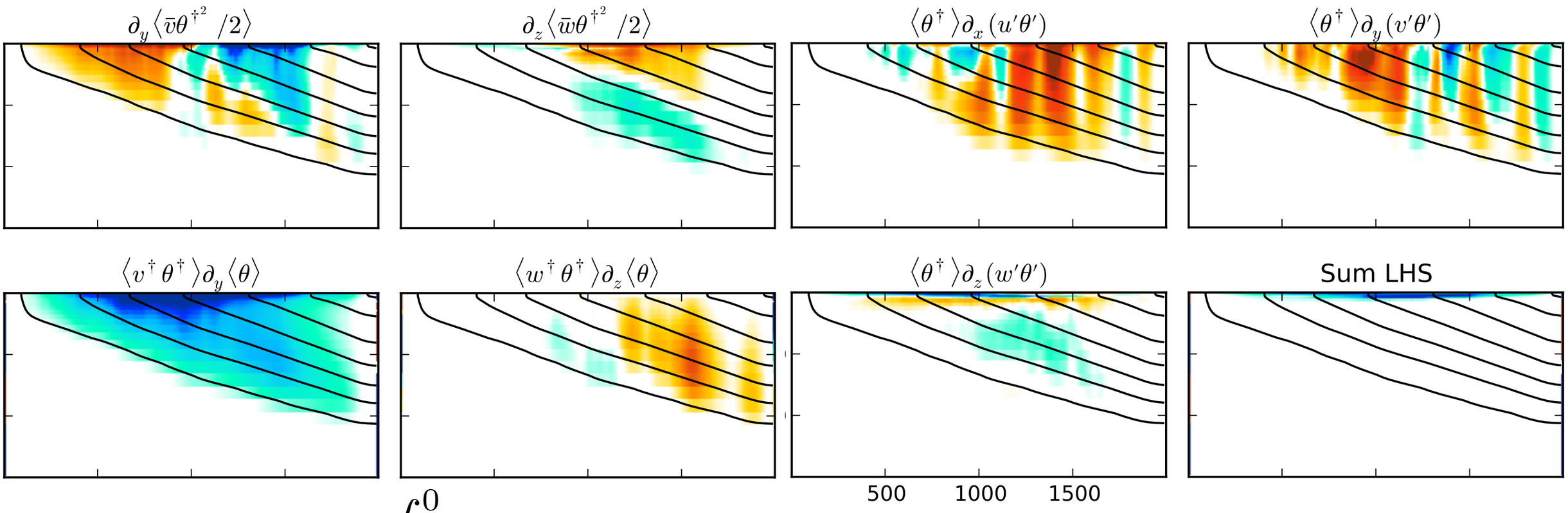
Why do Standing Eddies Transport Heat? Variance Budget for $\theta^\dagger$

$$\nabla \cdot \left(\langle u \rangle \frac{\langle \theta^{\dagger 2} \rangle}{2} + \frac{\langle u^\dagger \theta^{\dagger 2} \rangle}{2} \right) + \langle u^\dagger \theta^\dagger \rangle \cdot \nabla \langle \theta \rangle = -\langle \theta^\dagger \nabla \cdot (\overline{u'\theta'}) \rangle - \kappa_h \langle |\nabla \theta^\dagger|^2 \rangle - \langle \kappa_v |\theta^\dagger_z|^2 \rangle - \frac{\lambda}{\delta} \langle \theta^{\dagger 2}_s \rangle$$



Why do Standing Eddies Transport Heat? Variance Budget for $\theta^\dagger$

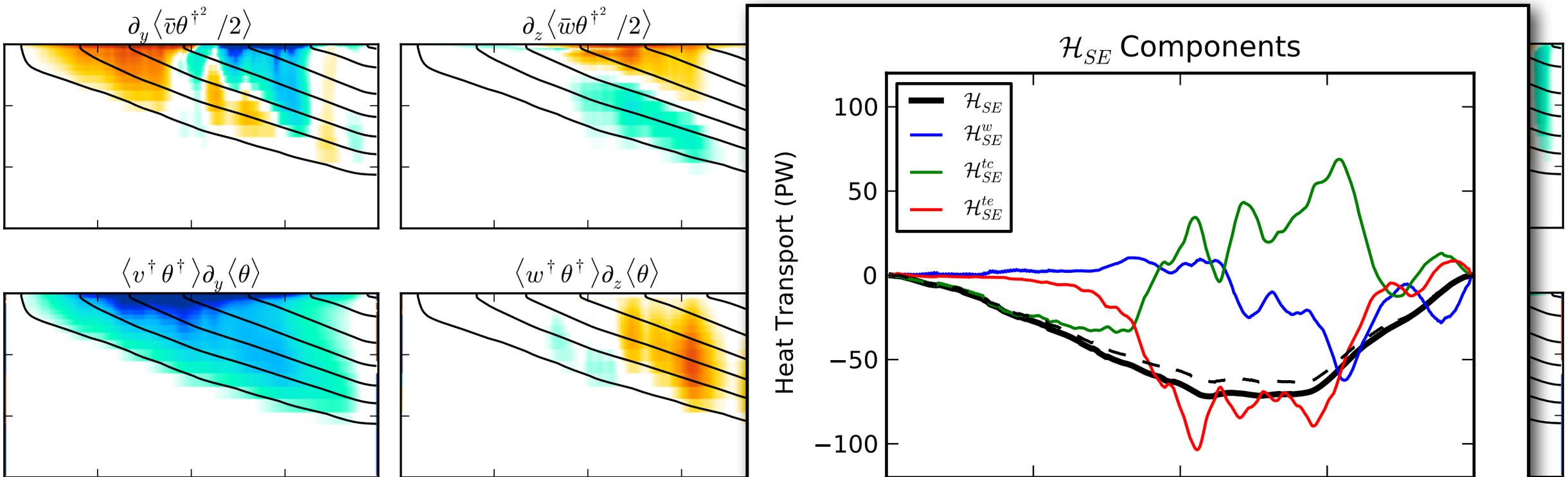
$$\nabla \cdot \left(\langle u \rangle \frac{\langle \theta^{\dagger 2} \rangle}{2} + \frac{\langle u^\dagger \theta^{\dagger 2} \rangle}{2} \right) + \langle u^\dagger \theta^\dagger \rangle \cdot \nabla \langle \theta \rangle = -\langle \theta^\dagger \nabla \cdot (\overline{u'\theta'}) \rangle - \kappa_h \langle |\nabla \theta^\dagger|^2 \rangle - \langle \kappa_v |\theta^\dagger_z|^2 \rangle - \frac{\lambda}{\delta} \langle \theta^{\dagger 2}_s \rangle$$



$$\begin{aligned}
\mathcal{H}_{SE} &= \rho_0 c_p L_x \int_{-H}^{0} \langle v^\dagger \theta^\dagger \rangle dz \\
&= -\rho_0 c_p L_x \int_{-H}^{0} \langle \theta \rangle_y^{-1} \left[\langle w^\dagger \theta^\dagger \rangle \langle \theta \rangle_z + \nabla \cdot \frac{\langle u^\dagger \theta^{\dagger 2} \rangle}{2} + \langle \theta^\dagger \nabla \cdot (\overline{u'\theta'}) \rangle \right] dz \\
&= \mathcal{H}^w_{SE} + \mathcal{H}^{tc}_{SE} + \mathcal{H}^{te}_{SE}
\end{aligned}$$

Why do Standing Eddies Transport Heat? Variance Budget for $\theta^\dagger$

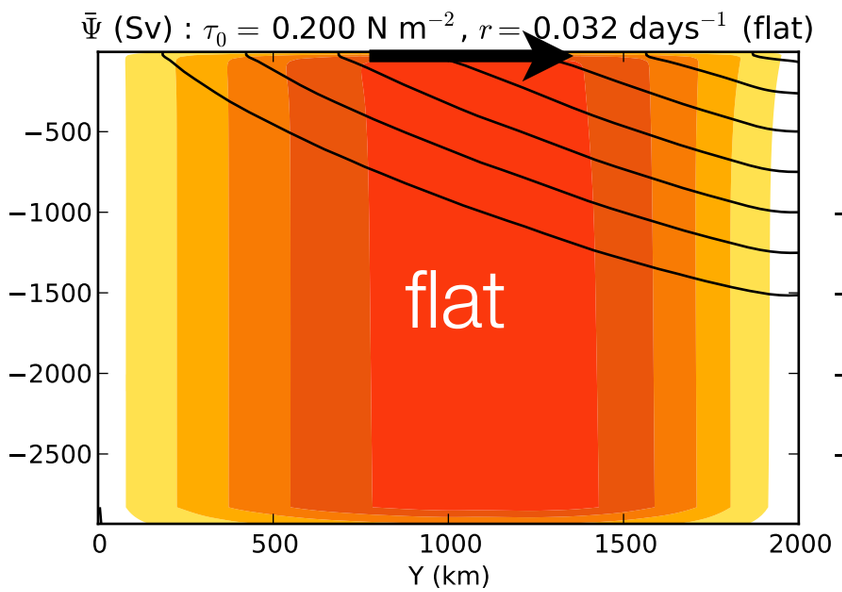
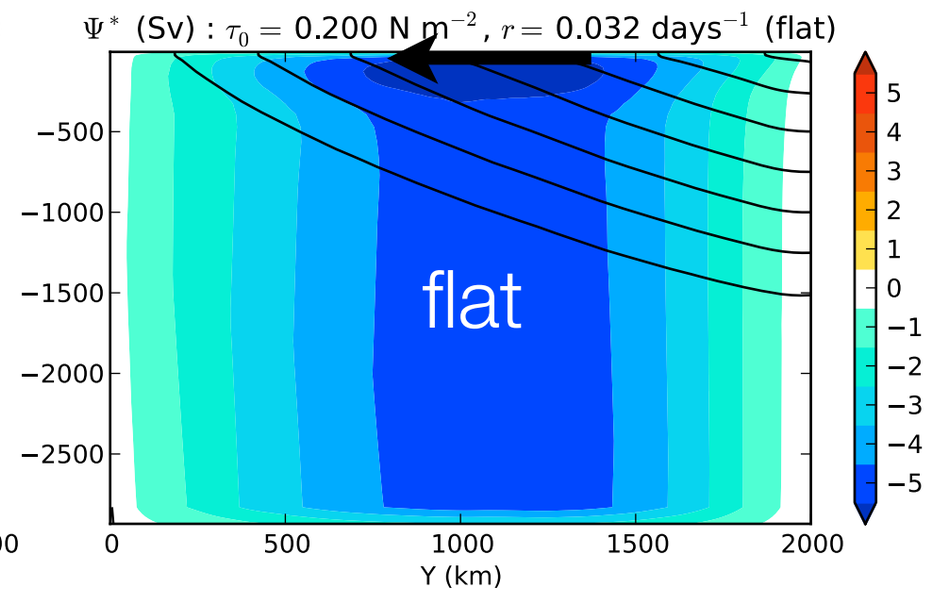
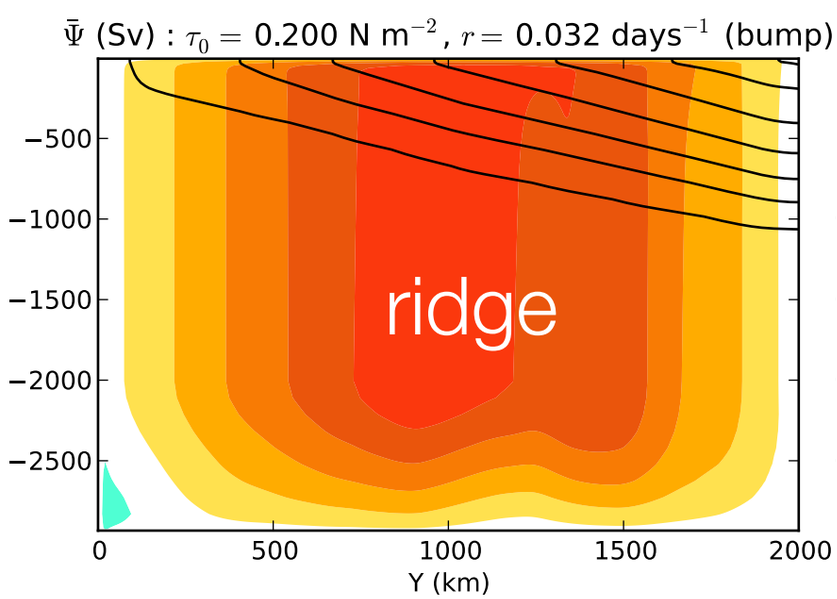
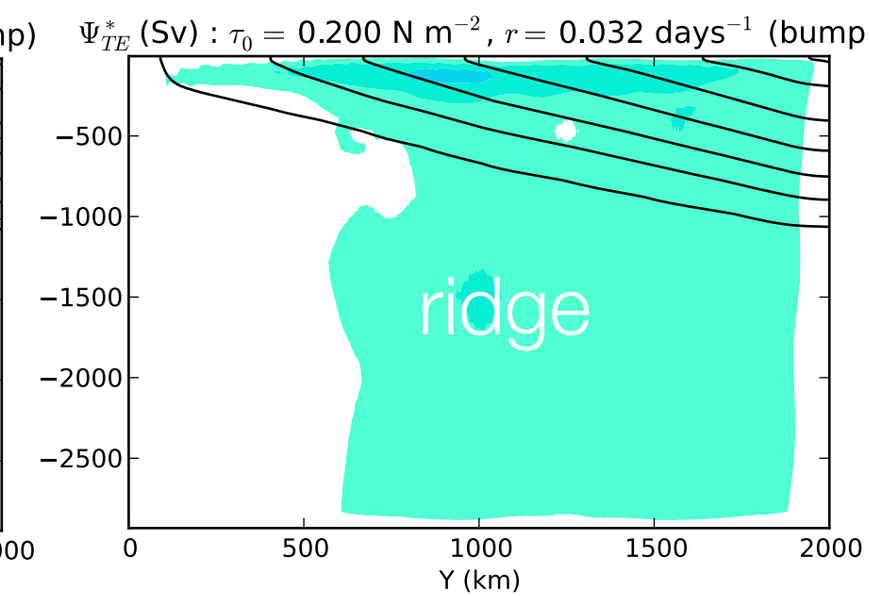
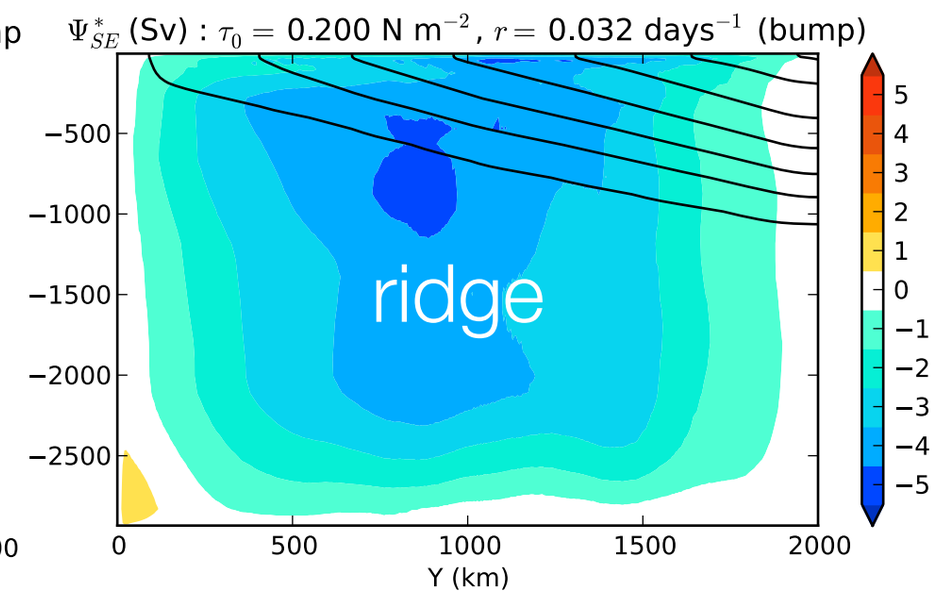
$$\nabla \cdot \left(\langle u \rangle \frac{\langle \theta^{\dagger 2} \rangle}{2} + \frac{\langle u^\dagger \theta^{\dagger 2} \rangle}{2} \right) + \langle u^\dagger \theta^\dagger \rangle \cdot \nabla \langle \theta \rangle = -\langle \theta^\dagger \nabla \cdot (\overline{u'\theta'}) \rangle - \kappa_h \langle |\nabla \theta^\dagger|^2 \rangle - \langle \kappa_v |\theta_z^\dagger|^2 \rangle - \frac{\lambda}{\delta} \langle \theta_s^{\dagger 2} \rangle$$



$$\begin{aligned}
\mathcal{H}_{SE} &= \rho_0 c_p L_x \int_{-H}^{0} \langle v^\dagger \theta^\dagger \rangle dz \\
&= -\rho_0 c_p L_x \int_{-H}^{0} \langle \theta \rangle_y^{-1} \left[\langle w^\dagger \theta^\dagger \rangle \langle \theta \rangle_z + \nabla \cdot \frac{\langle u^\dagger \theta^{\dagger 2} \rangle}{2} + \langle \theta^\dagger \nabla \cdot (\overline{u'\theta'}) \rangle \right] dz \\
&= \mathcal{H}_{SE}^{w} + \mathcal{H}_{SE}^{tc} + \mathcal{H}_{SE}^{te}
\end{aligned}$$

Meridional Overturning

Eulerian mean

Transient Eddy

Standing Eddy