SIO 214A Homework 1

Please hand in on paper, in class, on Thursday, October 7.

1.) A planet consisting of a uniform, solid core with mass M and radius R is surrounded by a fluid atmosphere in which the pressure is given by

$$p(\rho) = c^2\rho, \tag{1}$$ 1

where ρ is the mass density, and c^2 is a positive constant. The hydrostatic equation is

$$0 = -\frac{dp}{dr} + \rho F, \tag{2}$$ 2

where r is the radial coordinate, and F is the gravitational force per unit mass in the radial direction.

First, using the approximation

$$F \approx -g \equiv -\frac{GM}{R^2} \tag{3}$$ 3

determine the radial dependence, $p(r)$ and $\rho(r)$, of pressure and density.

In what sense can the *scale depth*

$$H_s \equiv \frac{c^2}{g} \tag{4}$$ 4

be regarded as the depth of the atmosphere?

Relate the constant of integration that appears in your solution to the total mass M_A of the atmosphere.

Repeat the calculation of $p(r)$ and $\rho(r)$ using the exact Newtonian gravity law,

$$F = -\frac{GM}{r^2} \tag{5}$$ 5

Show that the exact solution agrees with the approximate solution if $c^2/g << R$.

In both cases, assume that the planet is non-rotating, and that the atmosphere is at rest with respect to the planet. Neglect the gravitational self-attraction of the atmosphere.

2.) Calculate $p(r)$ and $\rho(r)$ for the case in which no solid core is present (planet consisting entirely of fluid). This time assume

$$p(\rho) = K\rho^2, \tag{6}$$ 6

where K is a constant. Use the exact Newtonian gravitation law, and take into account the gravitational self-attraction of the fluid. Show that, by a suitable rescaling of the variables, the hydrostatic equation can be brought into the form

$$\frac{d}{d\xi}\left(\xi^2 \frac{d\rho}{d\xi}\right) = -\xi^2 \rho(\xi) \tag{7}$$ 7

which has a solution

$$\rho(\xi) = \rho_0 \frac{\sin \xi}{\xi} \tag{8}$$ 8

where ρ_0 is a constant. Keeping in mind that ρ must be positive, is there a way to make sense of this solution?

Hints: Remember that a radially symmetric mass distribution within a spherical volume induces gravitational field outside that volume which is the same as if all the mass were concentrated at the center of the sphere.

Also, the mass of a spherical shell of thickness dr is $4\pi r^2 \rho(r) dr$.