

SHORTER CONTRIBUTION

Note on the Dynamics of the Antarctic Circumpolar Current¹

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Abstract

Unlike all other major ocean currents, the Antarctic Circumpolar Current probably does not have sufficient frictional stress applied at its lateral boundaries to balance the wind stress. The balancing stress is probably applied at the bottom, largely where the major submarine ridges lie in the path of the current. The meridional circulation provides a mechanism for extending the current to a large enough depth to make this possible.

Under the assumption that the wind stress on the sea surface is balanced by the frictional stress against the sides of the ocean basins one can arrive at a fairly satisfactory picture of the large scale circulation in various oceans (MUNK, 1950; MUNK and CARRIER, 1950). The computed transports differ at most by a factor of two from the observed transports. In the case of the Antarctic Circumpolar (AC) current a similar reasoning leads however to a computed transport one hundred times the observed transport.

In its simplest form we may regard the AC current as an eastward flow on a plane tangent to the earth at the South Pole. The flow is induced by constant eastward winds, and depends only on the distance r from the pole on this plane. In this system a balance between the wind stress τ and the lateral friction is expressed by

$$\tau + A \frac{d}{dr} \left(\frac{dM}{dr} + \frac{M}{r} \right) = 0$$

where A is the lateral kinematic eddy viscosity and M the eastward mass transport across a normal

vertical plane of unit width extending from surface to bottom.⁴ The solution which vanishes at the Antarctic continent ($r = r_0$) and at some other latitude ($r = r_1$) is

$$M = \frac{\tau}{3A} r (r - r_0) \left(\frac{r + r_0}{r_1 + r_0} \frac{r_1^2}{r^2} - 1 \right).$$

The total AC transport equals

$$\begin{aligned} T &= \int_{r_0}^{r_1} M dr = \\ &= \frac{\tau}{18A} \left(r_1^3 - r_0^3 - \frac{6 r_1^2 r_0^2}{r_1 + r_0} \ln \frac{r_1}{r_0} \right). \end{aligned}$$

Setting $\tau = 2$ dynes cm^{-2} , $A = 10^8$ $\text{cm}^2 \text{sec}^{-1}$, and placing the boundaries at 70° S and 45° S latitudes gives $T = 5 \times 10^{16}$ g sec^{-1} . At the narrowest section, Drake Passage, the boundaries are at 65° S and 55° S latitudes, and $T = 10^{15}$ g sec^{-1} . The observed transport is 10^{14} g sec^{-1} . The discrepancy is not materially altered by more elaborate calculations involving spherical coordinates and allowing for a variation of wind with latitude.

¹ Contribution from the Scripps Institution of Oceanography, New Series No. 515. This work has been supported by a contract with the Office of Naval Research.

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⁴ Strictly speaking we should replace the relative transport M by the absolute transport $M + \omega r h$, where ω is the earth's angular velocity and h the ocean depth. However, the second term which represents solid rotation drops out upon differentiation.

Among the possible explanations for the discrepancy are:

1. The value of A pertaining to the AC may have been taken too small, and should possibly be one to two orders of magnitude larger than $10^8 \text{ cm}^2 \text{ sec}^{-1}$. Various methods for other large scale ocean currents have led to values from 10^7 to $10^8 \text{ cm}^2 \text{ sec}^{-1}$.

2. The wind stress is balanced by stress against the bottom, rather than against the lateral boundaries as has been assumed. Somewhat artificially we may divide this bottom effect into (2a) skin friction, and (2b) the mountain effect.

2a. A bottom velocity so great that the wind stress is balanced entirely by skin friction, is given by SVERDRUP et al. (1942, p. 480),

$$v_b = 2.5 \sqrt{\frac{\tau}{\rho}} \ln \left(\frac{z + z_0}{z_0} \right).$$

Setting $\tau = 2 \text{ dynes cm}^{-2}$, $\rho = 1.0 \text{ g cm}^{-3}$, $z_0 = 2 \text{ cm}$, and $z = 100 \text{ cm}$ gives $v_b = 14 \text{ cm sec}^{-1}$ at 100 cm above bottom. This is approximately the maximum surface velocity. The value for the "roughness length" z_0 is based on unpublished measurements by Revelle and Fleming in San Diego Harbor. But this process could at best lead to a linear velocity gradient between top and bottom of $14 \text{ cm sec}^{-1} / 4 \text{ km} = 3.5 \cdot 10^{-5} \text{ sec}^{-1}$. In order for the resulting vertical stress, $3.5 \cdot 10^{-5} A_b$, to equal a wind stress of 2 dynes cm^{-2} at the surface the vertical eddy viscosity A_z would have to equal $6 \cdot 10^4 \text{ g cm}^{-1} \text{ sec}^{-1}$. Reported values are generally less than $2 \cdot 10^3 \text{ g cm}^{-1} \text{ sec}^{-1}$ in the upper few hundred meters, and even less at greater depth (SVERDRUP et al. 1942, p. 482).

2b. The foregoing equation refers to the statistical effect associated with "small" irregularities in the sea bottom. In meteorology, the mountain effect associated with the few largest irregularities on the earth's surface has been found at least as effective as skin friction (WIDGER, 1949), but in oceanography the mountain effect has been neglected. Let dh/dx denote the eastward (down current) slope of the sea bottom. The eastward component of force exerted on an element of bottom of area ds equals

$$p_b (dh/dx) ds,$$

where p_b is the pressure along the bottom. For small slopes, ds can be taken as the area projected on a horizontal plane. The net effect around a circle of latitude of circumference C vanishes if the pressure is constant ($\int dh = 0$), or if the depth is

constant ($dh/dx = 0$). If however the bottom pressure on the up current side exceeds on the average the pressure at the same level on the lee side by Δp_b , then the stress has a mean value of the order of

$$\frac{1}{C} \Delta p_b Ah$$

which could balance the mean wind stress. Here Ah is the sum of the heights of the sea mounts. At 60° S the current encounters the Kerguelen ridge (3 km), the Macquarie ridge (2 km), the South Pacific ridge (1 km), and the South Antillean arc (4 km). The currents must diminish over each ridge and accelerate between ridges, but on the average the mountain effect may balance the wind stress. Setting $Ah = 10 \text{ km}$, $C = 18,000 \text{ km}$, $\tau = 2 \text{ dynes cm}^{-2}$ gives $\Delta p_b \approx 4,000 \text{ dynes cm}^{-2} = 4 \text{ dynamic centimeters}$. At the surface the horizontal variations in pressure are of the order of a dynamic meter, with the larger values occurring on the up current side. The wind stress can therefore be accounted for by horizontal pressure gradients at the bottom similar in phase to those observed near the surface, and amounting to a few percent of the surface gradients.

If the wind stress is to be balanced by stresses along the sea bottom, the AC current must be deep enough to reach to the bottom. Sverdrup has pointed out that the sharp leftward displacement of the surface current over the submarine ridges would indicate that this is the case (SVERDRUP et al, 1942, pp. 466—469). We may consider two possible explanations:

1. Each layer induces, by turbulent interchange of momentum, motion in the layer beneath, and in this manner the wind stress is transmitted to the sea bottom. Because the fetch of the wind is essentially infinite this mechanism is more effective in the AC current than elsewhere.

2. The meridional circulation exports absolute angular momentum in the upper layer, and imports it in a lower layer. Suppose the loss of angular momentum in the upper layer alone associated with a flow of $Q \text{ g sec}^{-1}$ northward across the 45° S parallel of latitude is balanced entirely by the wind torque over the ocean between the Antarctic continent (70° S) and 45° S :

$$\begin{aligned} Q (\omega R \cos 45^\circ) (R \cos 45^\circ) &= \\ &= \int_{45^\circ \text{ S}}^{70^\circ \text{ S}} \tau (2 \pi R^2 \cos \varphi) (R \cos \varphi) d \varphi, \end{aligned}$$

where ω is the earth's angular velocity and R its

radius. Setting $\tau = 2$ dynes cm^{-2} leads to $Q = 3 \times 10^{13}$ g sec^{-1} . This value is in agreement with the observed northward transport in the upper 500—1,000 meters, due mostly to the Benguela and Peru Currents. If we suppose for the moment that the difference between precipitation and evaporation over the whole region south of the latitude 45° S and in addition a possible decrease of the Antarctic ice cap could provide all this water, then the wind stress could be accounted for by the export of absolute angular momentum into the southern oceans. Actually not more than about 0.3–0.5 per cent of the above value of Q could be provided in this way⁵, so that virtually all of the water lost in the upper layers must be returned to the Antarctic Sea at a lower level⁶, apparently at depths between 1,000 and 4,000 meters. Conservation of angular momentum then requires an increasing cyclonic (eastward) circulation at these great depths with decreasing distance from the pole. Deacon's analysis of the *DISCOVERY* observations in fact shows a remarkably uniform horizontal density gradient throughout most of the water column, whereas, for example, Iselin's sections across the Gulf Stream reveal a sharp decrease in horizontal gradients at a thousand meters (compare figs. 159 and 186, SVERDRUP et al. 1942). From this point of view the principal action of the wind is not so much to induce cyclonic circulation throughout the water column, as to prevent anticyclonic circulation of the northward drifting surface waters.

To summarize: deep water in the Antarctic ocean drifting towards the axis of the rotating earth develops an easterly (relative) flow. The retarding pressure of the submarine ridges against the deep current balances the stress exerted by the wind on the water's surface.

⁵ Provided a reported secular rise in world-wide sea level by 1 mm per year is due alone to melting in the Antarctic.

⁶ An additional complication is the northward flow along the bottom of the heavy water formed near the Antarctic continent during freezing, and accounting for most of the bottom water in deep basins the world over.

The foregoing speculation regarding the dynamics of the AC current would make it differ radically from other major ocean current systems, which are limited essentially to the upper 1,000 meters, and for which the stresses against the lateral boundaries are adequate to balance the wind stress. The difference is due largely to boundaries which prevent the zonal flow of water around the earth. Assume that the Drake Passage should close, and that the Macquarie Ridge should emerge from the surface. Then the water in the confined sector would pile up against the downwind boundary. If the wind blows with equal strength at all latitudes, the resulting horizontal pressure gradient will balance the wind stress and there would be no circulation involving a horizontal gyre. If however the winds at higher latitudes are weaker or, as is the case, reverse in direction, the resulting wind torque would set up a gyre within this confined sector of the Antarctic Ocean. As an example, the Gulf Stream is the western component of a "subtropical gyre" maintained by the easterly winds to the south and the westerly winds to the north. In a confined ocean the transport depends therefore on the curl of the wind stress (MUNK, 1950); in a zonal (Antarctic) ocean the transport has been shown here to be proportional to the wind stress itself.

We have also shown the transport in a zonal ocean to vary as A^{-1} provided the wind stress is balanced by lateral friction. In a confined ocean where such a balance does in fact exist the transport is independent of A (MUNK, 1950)! This surprising result has to do with the west-east asymmetry induced by the earth's rotation on a confined gyre. The center of the gyre (Sargasso Sea) is displaced to the west, and the western current (Gulf Stream) is pressed against the boundary. If A is very large, this asymmetry, and hence the shear against the western boundary, are small. If A is relatively small the asymmetry is very pronounced, and the shear very large. The product of A times the shear, i. e., the lateral friction, remains the same, and is so large in this western region that it almost balances the wind stress over the entire subtropical gyre.

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