SIO 203C: Partial Differential Equations

Lecture notes by Rick Salmon

Spring quarter 2001

1. Review of some elementary methods applied to the heat equation.
Separation of variables, Fourier analysis, Green's functions, symmetry. Navigating from one Green's function to another. Method of images. Similarity solutions. Dimensional analysis and symmetry groups.

2. First-order linear equations.
Characteristics. Well-posedness. Cauchy problem for first-order equations. Fredholm alternative.

3. First-order equations as a limiting case of second-order equations.
The advection-diffusion equation. Subcharacteristics, boundary layers, internal boundary layers.

4. Burger's equation.
First-order quasilinear equations. Boundary layer theory. Shock fitting. The Cole-Hopf transformation.

5. Classification of second-order equations.
Reduction to canonical form. Analogy with quadratic forms. Proof of the spectral decomposition theorem.

6. The wave equation.
D'Alembert's solution. Propagation of singularities. General initial value problem in 1 and 3 dimensions. Descent to 2 dimensions. Balloon break.

7. Elliptic equations; variational principles.
Elliptic equations as minimization problems. The finite element method. Boundary conditions. Helmholtz equation.

8. The analogy between matrices and operators.
Self-adjointness. Completeness.

9. Classification of higher-order quasi-linear equations as systems of first-order equations.
Hyperbolic systems. Examples from shallow-water theory.