Interaction of near-inertial waves with geostrophic turbulence
(in the ocean)

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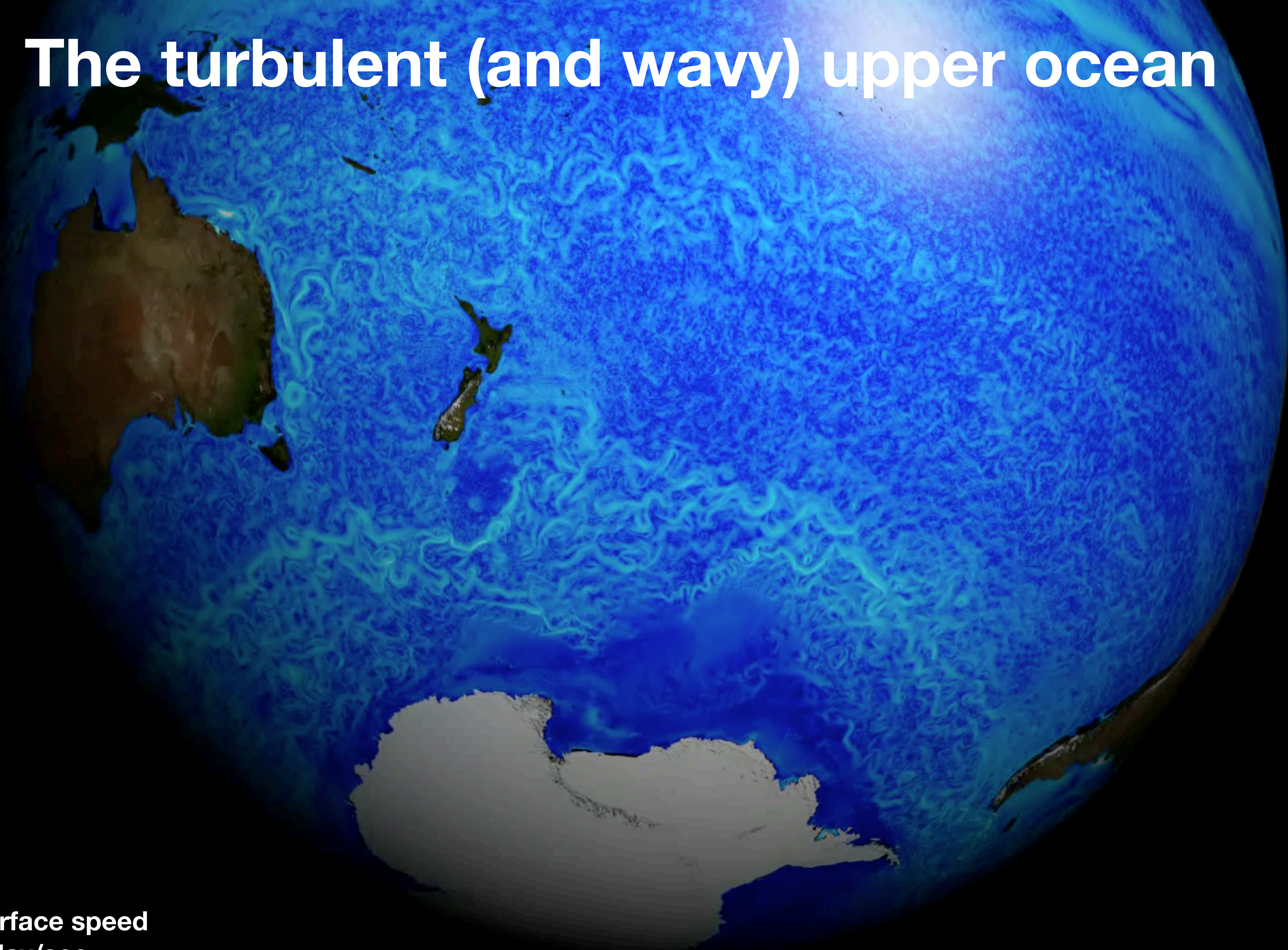
Congratulations Triantaphyllos!

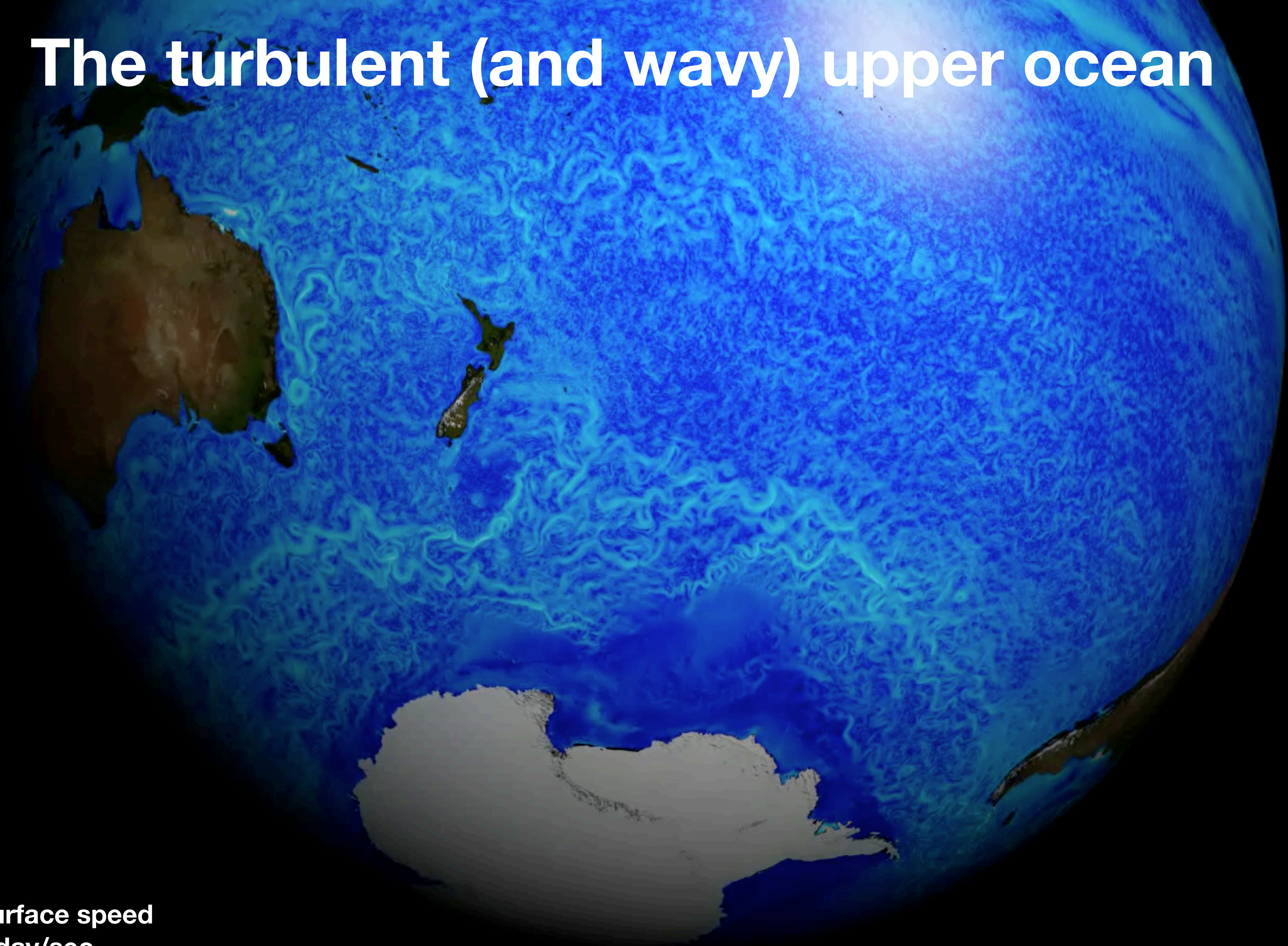
The turbulent (and wavy) upper ocean



Surface speed
1 day/sec

Chris Henze, NASA

The turbulent (and wavy) upper ocean



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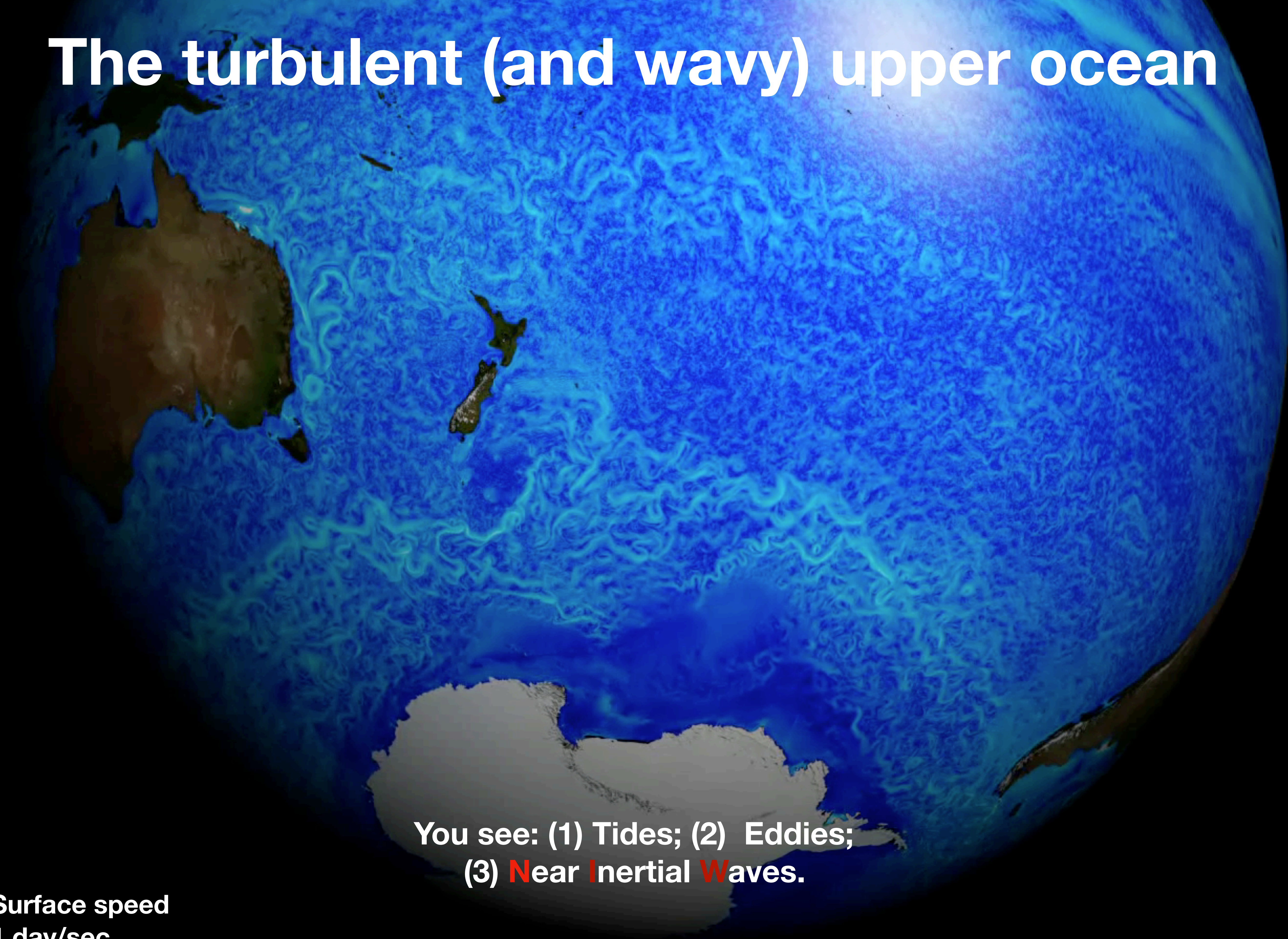
The turbulent (and wavy) upper ocean

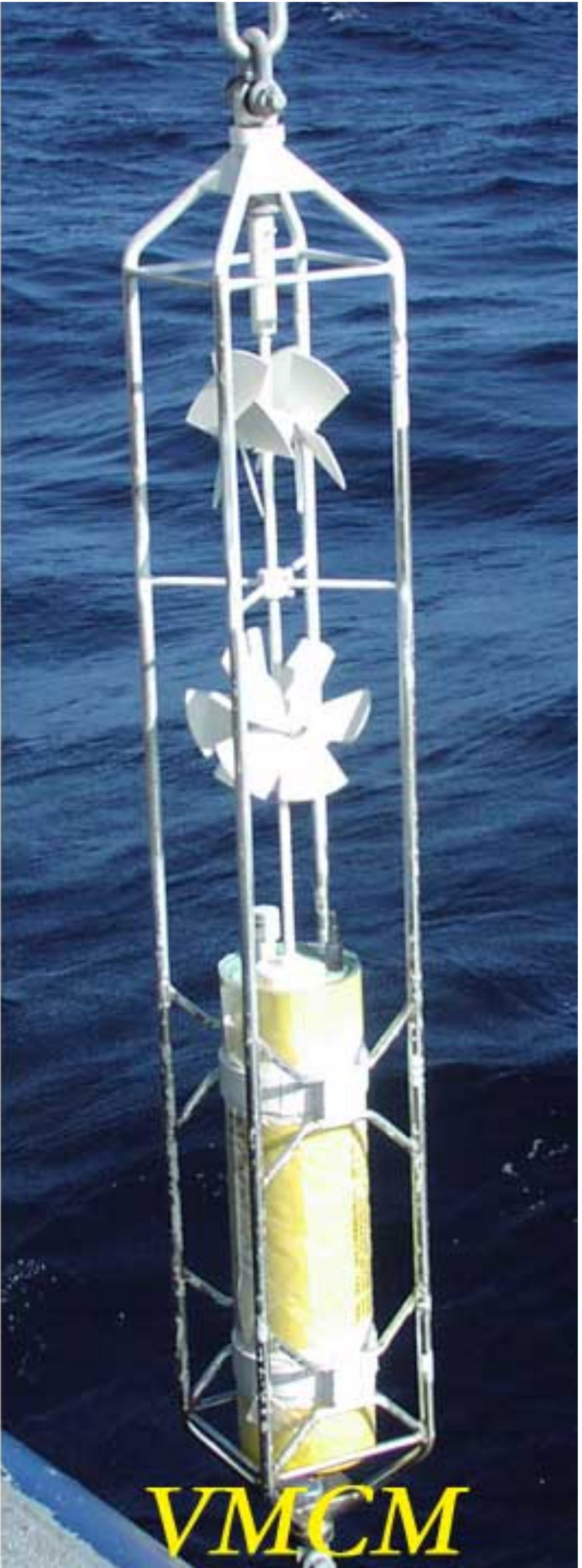


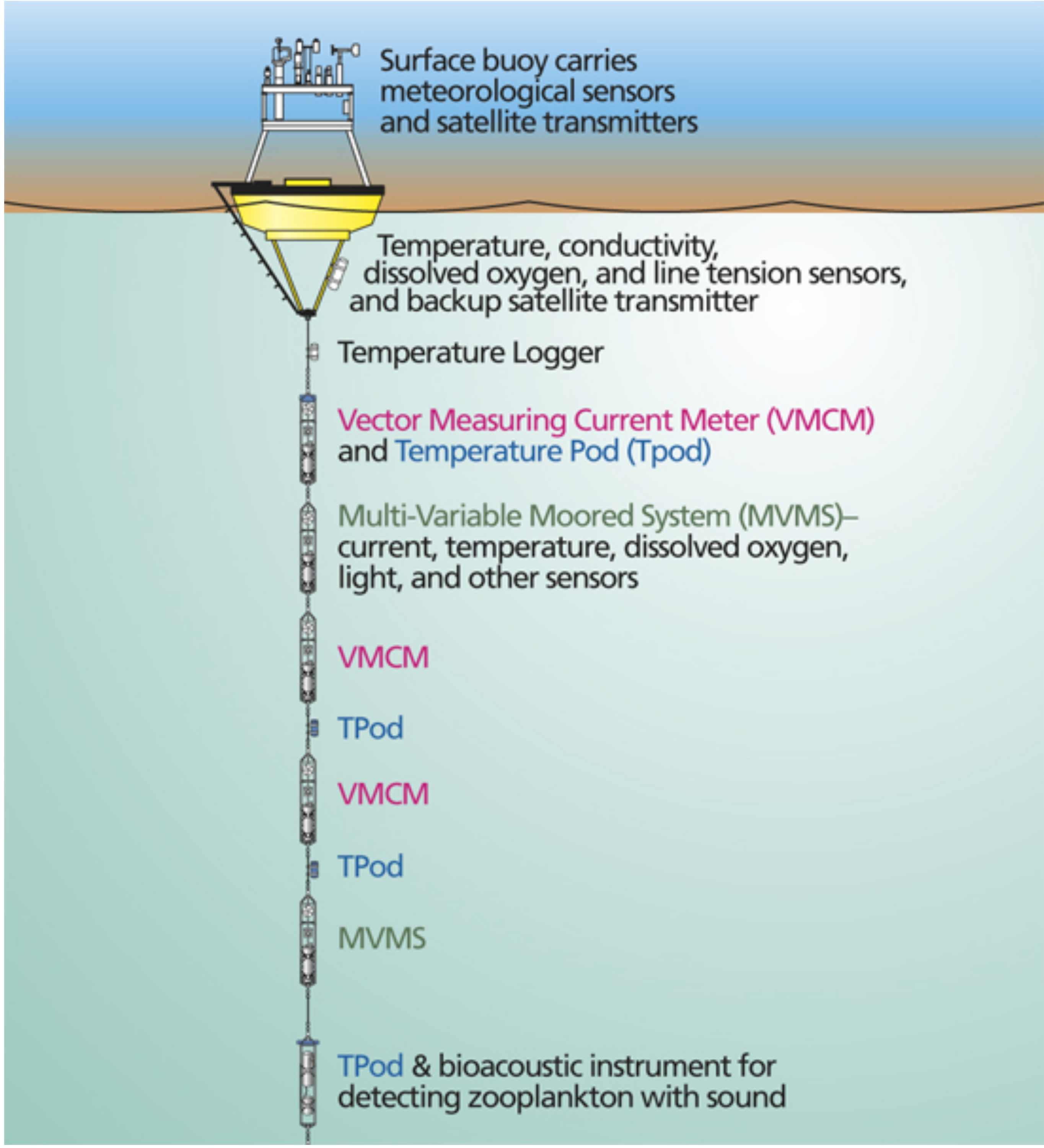
You see: (1) Tides; (2) Eddies;
(3) Near Inertial Waves.

Surface speed
1 day/sec

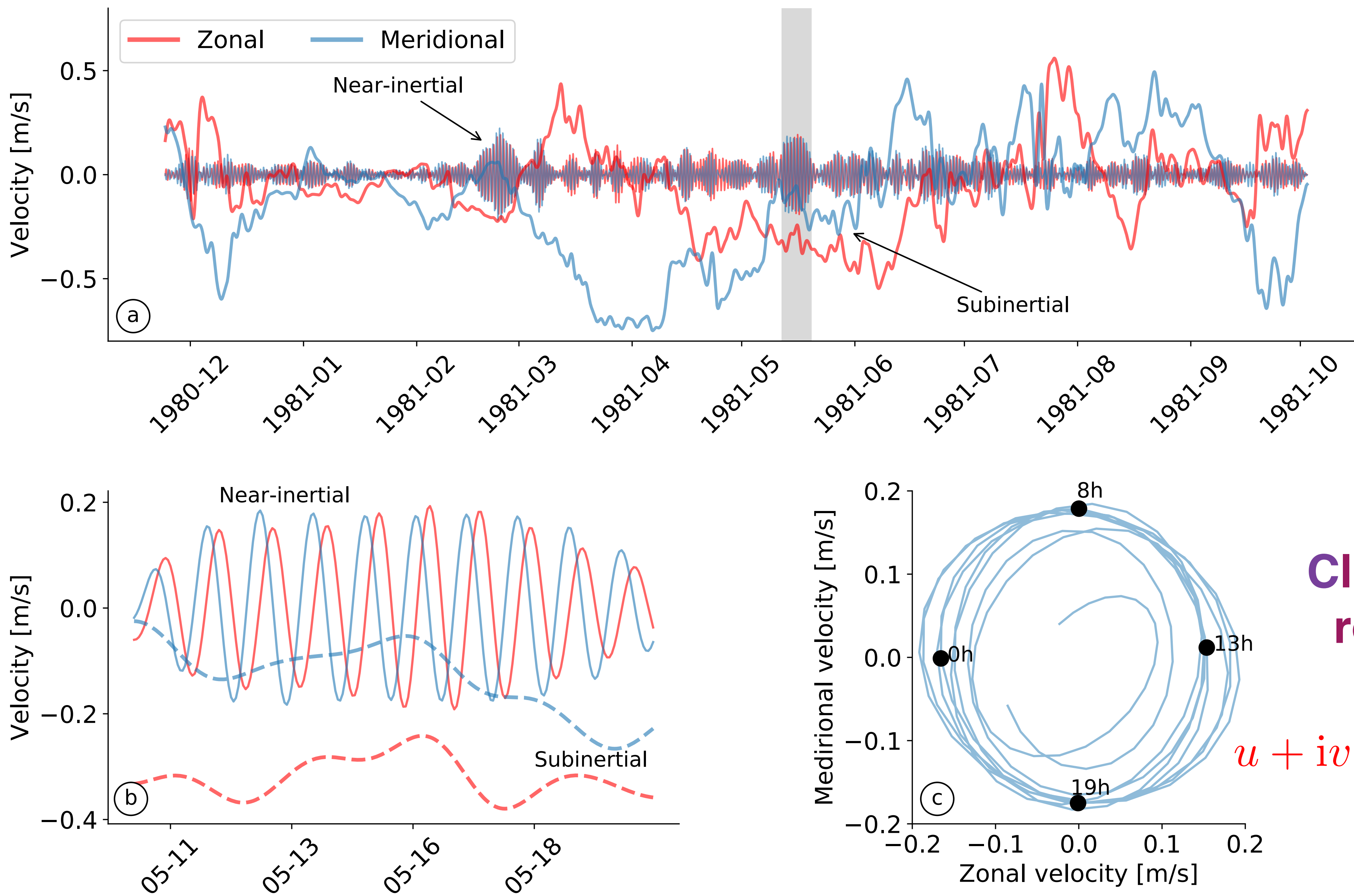
Chris Henze, NASA

Look at measurements of the horizontal velocity taken by a current meter at a single point in the ocean.





Time series from WHOI mooring 699
260m depth, near the Kuroshio in the NP



Clockwise rotation

$u + iv$

Eddies and waves are well separated in frequency (but not in space).

Motion in the Ocean

Eddies and waves are well separated in frequency.

There is no well defined spatial separation.

The near-inertial peak contains about half of the IGW energy and most of the vertical shear.

The near-inertial peak is

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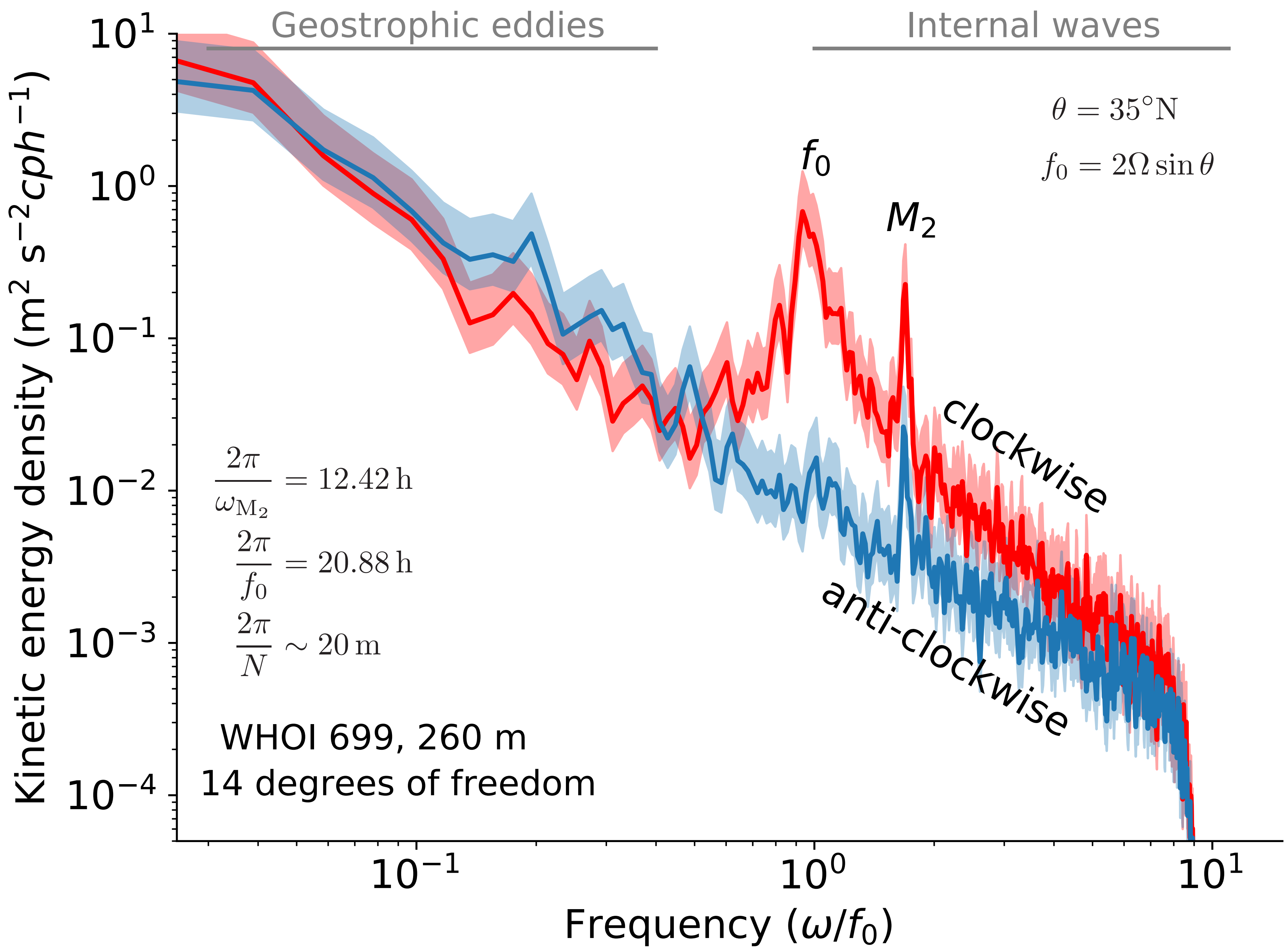
$$0.8 < \frac{\omega}{f_0} < 1.2$$



NIWs are wind forced

NIWs are wind forced, intermittent and volatile.

NIWs are not part of the "universal IGW continuum".

The radius of an "inertial circle" is:

$$R = \frac{\tilde{U}}{f}$$

$$= \frac{0.6\,\mathrm{MKS}}{10^{-4}\,\mathrm{MKS}} = 6\mathrm{km}$$

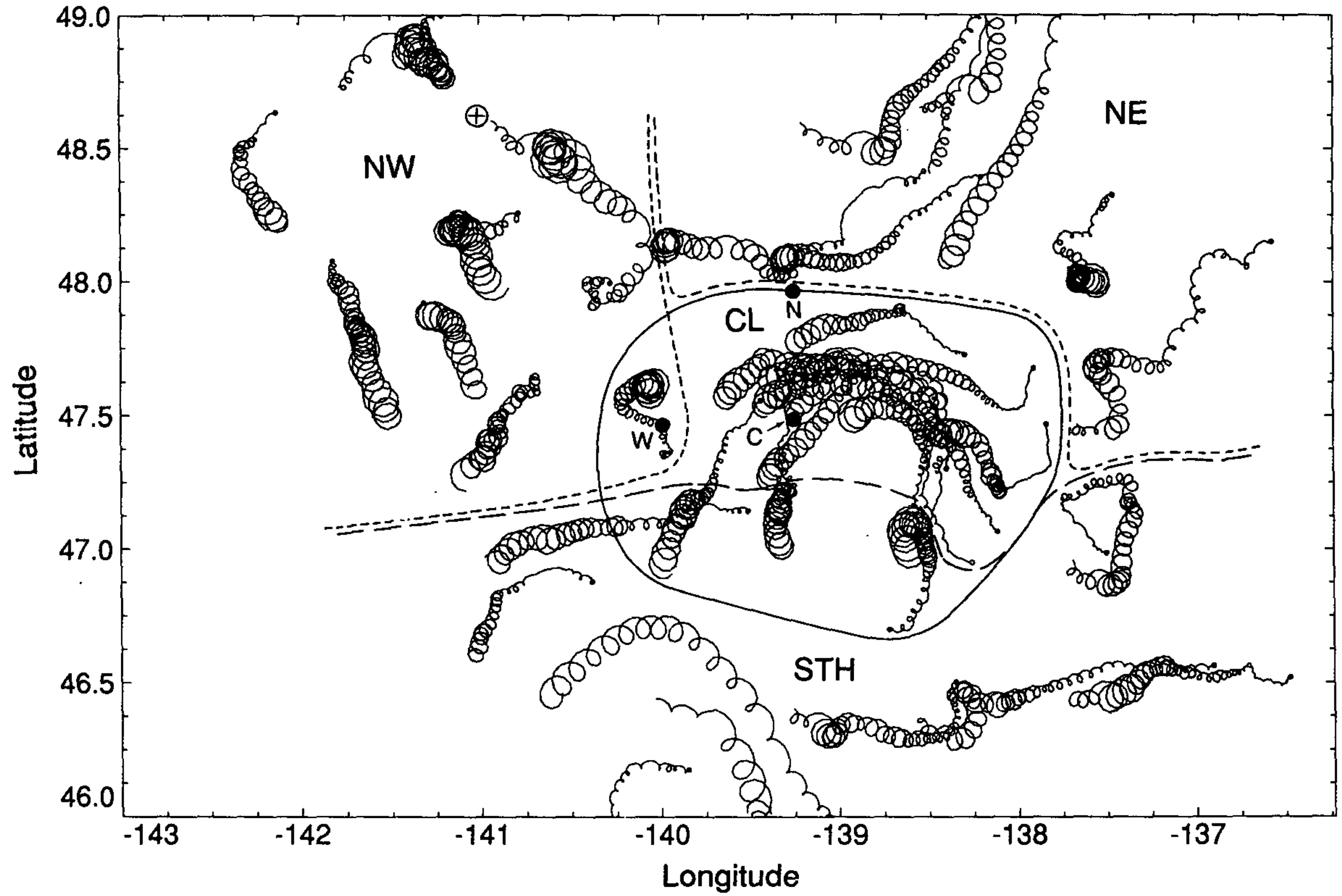


FIG. 1. Mixed layer drifter trajectories for days 275–300 of 1987 interpolated and filtered as described in the text. Many drifters were not deployed until day 280. Moorings (N, W, C) are indicated by the three large dots. The light lines define the geographic subregions used in the analysis: NE, NW (small dashes); CL (solid line); and STH (long dashes). The large circled "+" indicates a sample drifter discussed in the text.

Ocean Storms Experiment, 1987, North Pacific

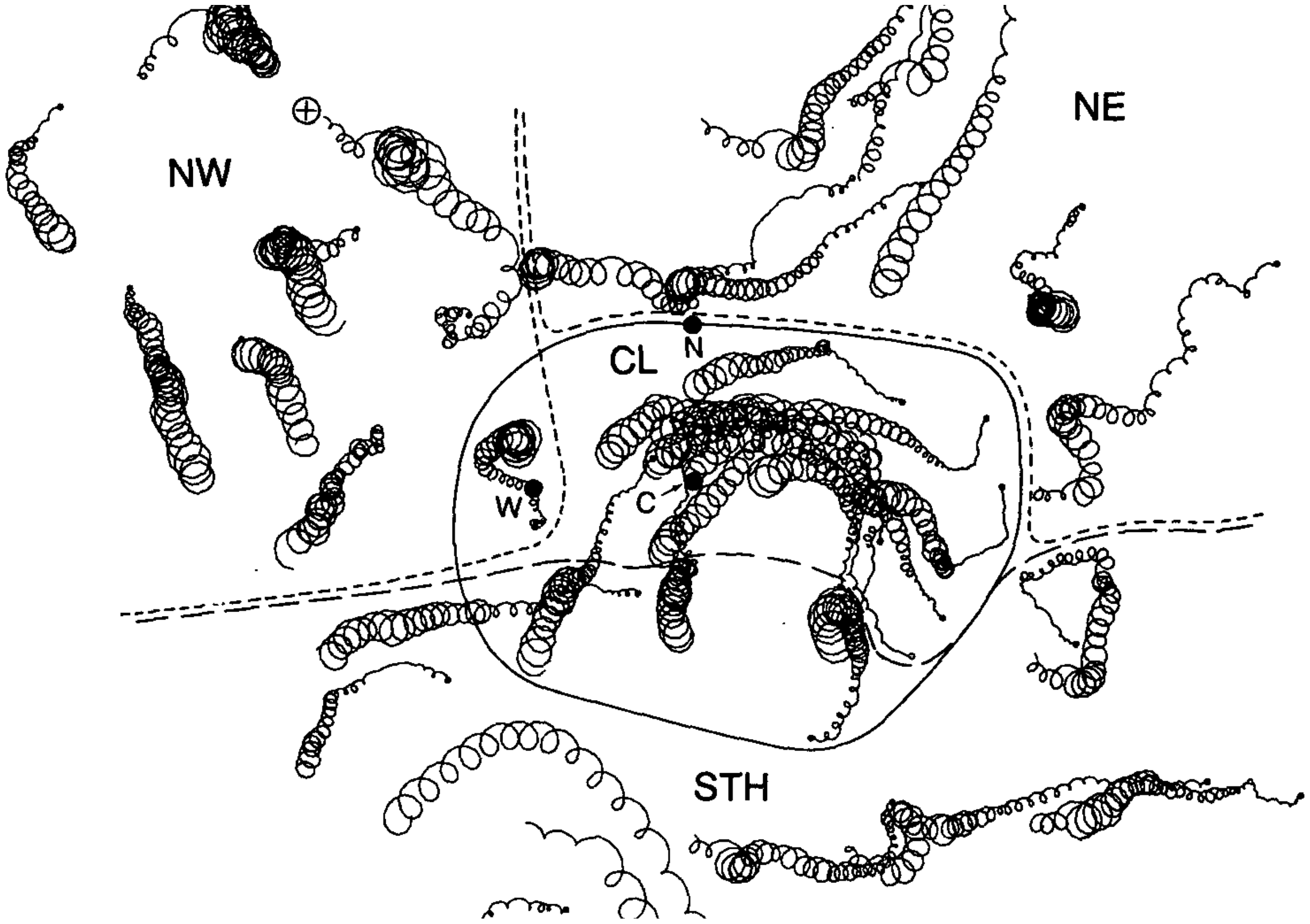
Near-Inertial Waves

The leading-order balance in the horizontal momentum equations is:

$$u_t - fv \approx 0$$
$$v_t + fu \approx 0$$
$$\Rightarrow (u + \mathrm{i}v)_t + \mathrm{i}f(u + \mathrm{i}v) \approx 0$$

In terms of the back-rotated velocity:

$$\partial_t \left[\mathrm{e}^{\mathrm{i}ft}(u + \mathrm{i}v) \right] \approx 0$$



In the shallow (50-100m deep) surface layer NIWs are horizontally coherent over separations greater than 1000km.

Deeper down, horizontal coherence scales are ~ 50km-100km.

The radius of an "inertial circle" is:

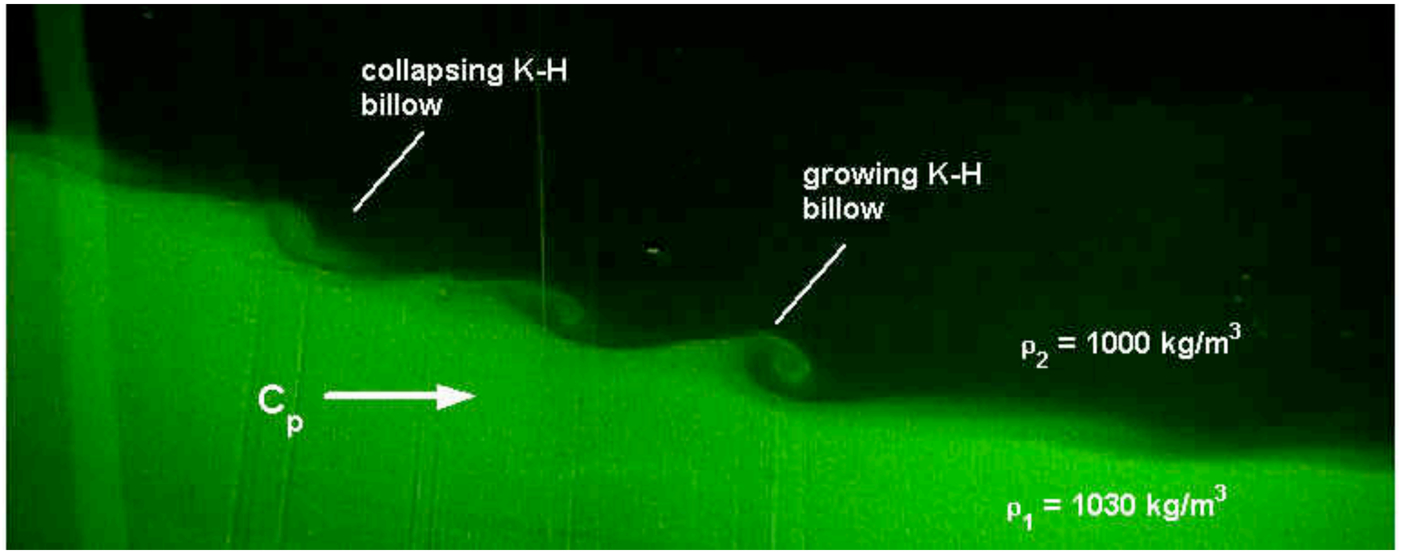
$$R = \frac{\tilde{U}}{f} = \frac{0.6\,\mathrm{MKS}}{10^{-4}\,\mathrm{MKS}} = 6\mathrm{km}$$

The role of NIWs in oceanography

Aren't NIWs just a very special case of IGWs?

$$f \leq \omega_{IGW} = \sqrt{\frac{N^2k^2 + f^2m^2}{k^2 + m^2}} \leq N$$

NIWs contain most of the vertical shear in the ocean and therefore control mixing.



$$\mathrm{Ri} = \frac{N^2}{u_z^2 + v_z^2}$$

The leading-order NIW dynamical balance is trivial. Therefore many small physical processes are responsible for NIW evolution.

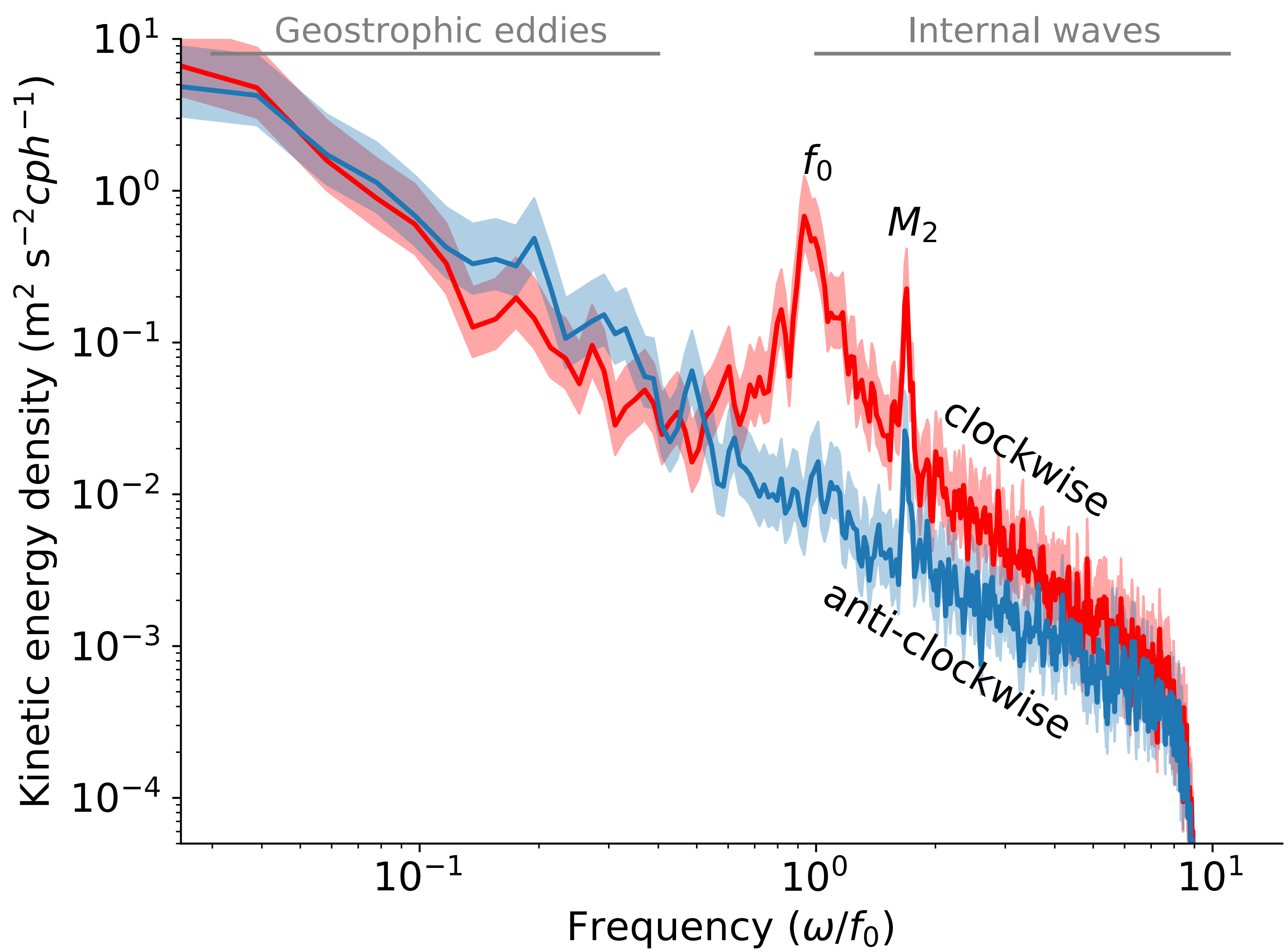
$$\partial_t(u + \mathrm{i}v) + f(u + \mathrm{i}v) \approx 0$$

Energy exchange between balanced flow and IGWs is mostly due to NIWs.

$$Q_t + J(\psi, Q) = 0$$

$$Q = \Psi_{xx} + \Psi_{yy} + \left(\frac{f^2}{N^2}\Psi_z\right)_z$$

This concludes the general introduction



Now derive a phase-averaged description of NIW evolution.

This is an NIW analog of the QG approximation.

$$Q_t + J(\psi, Q) = 0$$

$$Q = \Psi_{xx} + \Psi_{yy} + \left(\frac{f^2}{N^2}\Psi_z\right)_z$$

The Answer

(Young & Ben Jelloul 1997)

The master variable is the back-rotated velocity $\mathsf{L}A$

$$u + \mathrm{i}v = \mathrm{e}^{-\mathrm{i}ft}\mathsf{L}\,A$$

The back-rotated velocity satisfies a phase-averaged evolution equation

$$\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A + \tfrac{\mathrm{i}}{2}f\triangle A = 0$$

Notation:

$$\Psi(x,y,z,t) = \text{QG streamfunction}\,,$$
$$\mathsf{L}A(x,y,z,t) = \text{the back-rotated velocity}$$

$$\mathrm{i}^2 = -1\,, \qquad \triangle \stackrel{\text{def}}{=} \partial_x^2 + \partial_y^2\,, \qquad \mathsf{L} \stackrel{\text{def}}{=} \partial_z \frac{f^2}{N^2}\partial_z$$

The QG PV uses the same two differential operators

$$Q = \triangle\Psi + \mathsf{L}\Psi$$

Preliminary considerations

Let's examine the dispersion relation of hydrostatic IGWs

$$(u, v, w, p, b) \propto \mathrm{e}^{\mathrm{i}kx+\mathrm{i}mz-\mathrm{i}\omega t}$$

$$\omega = \sqrt{f^2 + \frac{N^2k^2}{f^2m^2}} \quad \overset{\text{NIW}}{\Rightarrow} \quad \omega \approx f + \underbrace{\tfrac{1}{2}\frac{N^2k^2}{fm^2}}_{\ll f}$$

Note NIWs have extreme aspect ratio — like pancakes.

$$\frac{N}{f} \gg 1\,, \quad \text{but still} \quad \underbrace{\frac{N^2k^2}{f^2m^2} = \left(\frac{N\lambda_v}{f\lambda_h}\right)^2}_{\text{Burger number}} \ll 1$$

The Burger number is the order parameter in the following expansion. The Rossby number is secondary.

1. A phase-averaged NIW equation

Linearize around a quasigeostrophic flow

$$[U, V, W, B, P] = [-\Psi_y\,, \Psi_x\,, 0\,, f\Psi_z\,, f\Psi]$$

Use multiple time-scales. Avoid WKB.

$$\partial_t \mapsto \partial_t + \epsilon^2 \partial_{t_2}$$

$$\epsilon^2 = Bu = Ro$$

$$u_t + \epsilon^2 [u_{t_2} + \boldsymbol{U} \cdot \boldsymbol{\nabla} u + \boldsymbol{u} \cdot \boldsymbol{\nabla} U + p_x] - fv = 0\,,$$

$$v_t + \epsilon^2 [v_{t_2} + \boldsymbol{U} \cdot \boldsymbol{\nabla} u + \boldsymbol{u} \cdot \boldsymbol{\nabla} U + p_y] + fu = 0\,,$$

$$p_z + b = 0\,,$$

$$u_x + v_y + w_z = 0\,,$$

$$b_t + \epsilon^2 [b_{t_2} + \boldsymbol{U} \cdot \boldsymbol{\nabla} b + \boldsymbol{u} \cdot \boldsymbol{\nabla} B] + wN^2 = 0\,.$$

The "pressureless" leading-order solution

$$\partial_s = \tfrac{1}{2}\left(\partial_x - \mathrm{i}\partial_y\right)$$

$$u_0 + \mathrm{i}v_0 = \left(\frac{f^2}{N^2} A_z\right)_z \mathrm{e}^{-\mathrm{i}ft_0}\,,$$

$$w_0 = -\frac{f^2}{N^2} A_{zs}\,\mathrm{e}^{-\mathrm{i}ft_0} + \mathrm{cc}\,.\,,$$

$$b_0 = \mathrm{i}fA_{zs}\,\mathrm{e}^{-\mathrm{i}ft_0} + \mathrm{cc}\,.\,,$$

$$p_0 = \mathrm{i}fA_s\,\mathrm{e}^{-\mathrm{i}ft_0} + \mathrm{cc}\,.\,,$$

2. A phase-averaged NIW equation

(Young & Ben Jelloul 1997)

There is a solvability condition at next order

$$(\partial_t + \mathrm{i}f)(u_2 + \mathrm{i}v_2) = \mathrm{e}^{+\mathrm{i}ft}[\cdots] - \mathrm{e}^{-\mathrm{i}ft}\underbrace{\left[\partial_{t_2}\mathsf{L}A + J(\Psi,\mathsf{L}A) + \tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A + \tfrac{\mathrm{i}}{2}f\,\triangle A\right]}_{\mapsto 0}$$

The NIW evolution equation

$$\partial_{t_2}\mathsf{L}A + J(\Psi,\mathsf{L}A) + \tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A + \tfrac{\mathrm{i}}{2}f\triangle A = 0$$

Physical interpretation

Advection:	$\boldsymbol{U}\cdot\boldsymbol{\nabla}(u,v)$	$\Rightarrow$	$J(\Psi,\mathsf{L}A)$
Refraction:	$\boldsymbol{u}\cdot\boldsymbol{\nabla}(U,V)$	$\Rightarrow$	$\tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A$
Dispersion:	$\boldsymbol{\nabla}p$	$\Rightarrow$	$\tfrac{\mathrm{i}}{2}f\,\triangle A$

Differential operators

$$\triangle \stackrel{\text{def}}{=} \partial_x^2 + \partial_y^2\,, \qquad \mathsf{L} \stackrel{\text{def}}{=} \partial_z \frac{f^2}{N^2}\partial_z$$

Special solutions

First, Ψ=0:

$$\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A + \tfrac{\mathrm{i}}{2} f \triangle A = 0$$

Then we recover the NIW dispersion relation.

$$\omega \approx f + \tfrac{1}{2}\frac{N^2 k^2}{f m^2}$$

$$A = \mathrm{e}^{\mathrm{i}kx + \mathrm{i}mz - \mathrm{i}\sigma t},$$

$$\Rightarrow \qquad \sigma = \frac{N^2 k^2}{2 f m^2}$$

Advection is intuitive.

$$J(\Psi, \mathsf{L}A)$$

How about refraction term?

$$\tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A$$

$$\triangle \stackrel{\mathrm{def}}{=} \partial_x^2 + \partial_y^2\,, \qquad \mathsf{L} \stackrel{\mathrm{def}}{=} \partial_z \tfrac{f^2}{N^2} \partial_z$$

Refraction $\frac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A$

An elliptical eddy

$$U = -\Psi_y = -\alpha y\,, \qquad V = \Psi_x = \beta x$$
$$\Rightarrow \quad \triangle\Psi = \alpha + \beta$$

A special solution:
a large scale wave.

$$u_t - \alpha y u_x + \beta x u_x - (f+\alpha)v + p_x = 0\,,$$
$$v_t - \alpha y v_x + \beta x v_x + (f+\beta)u + p_y = 0\,.$$

The frequency shift is:

$$\omega = \sqrt{(f+\alpha)(f+\beta)} \approx f + \tfrac{1}{2}(\alpha+\beta)$$

(Small Rossby number)

The corresponding solution of the phase-averaged equation is:

$$\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A + \tfrac{\mathrm{i}}{2} f \triangle A = 0$$
$$\Rightarrow u + \mathrm{i}v = \exp\left[-\mathrm{i}ft - \tfrac{\mathrm{i}}{2}(\alpha+\beta)t\right]$$

So the “effective inertial frequency is:

$$f_{\mathrm{eff}} = f + \tfrac{1}{2}\triangle\Psi$$

The (flawed) 2D quantum analogy

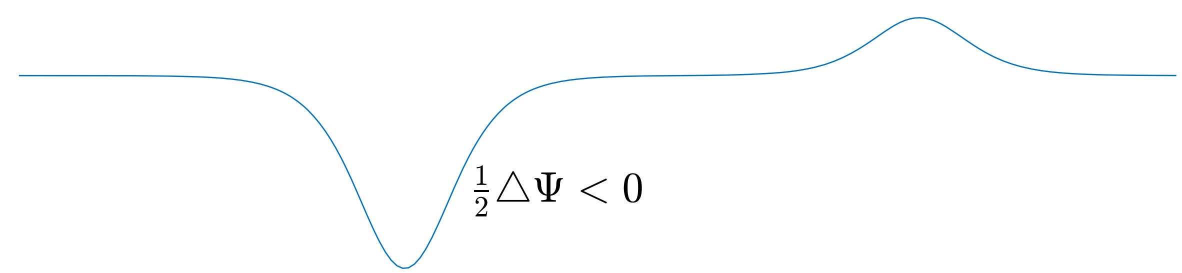
A vertical plane wave solution:

$$A \mapsto \mathrm{e}^{\mathrm{i}mz} A(x,y,t)\,,$$

$$\mathsf{L} \mapsto -\left(\frac{mf}{N}\right)^2 = -m'^2$$

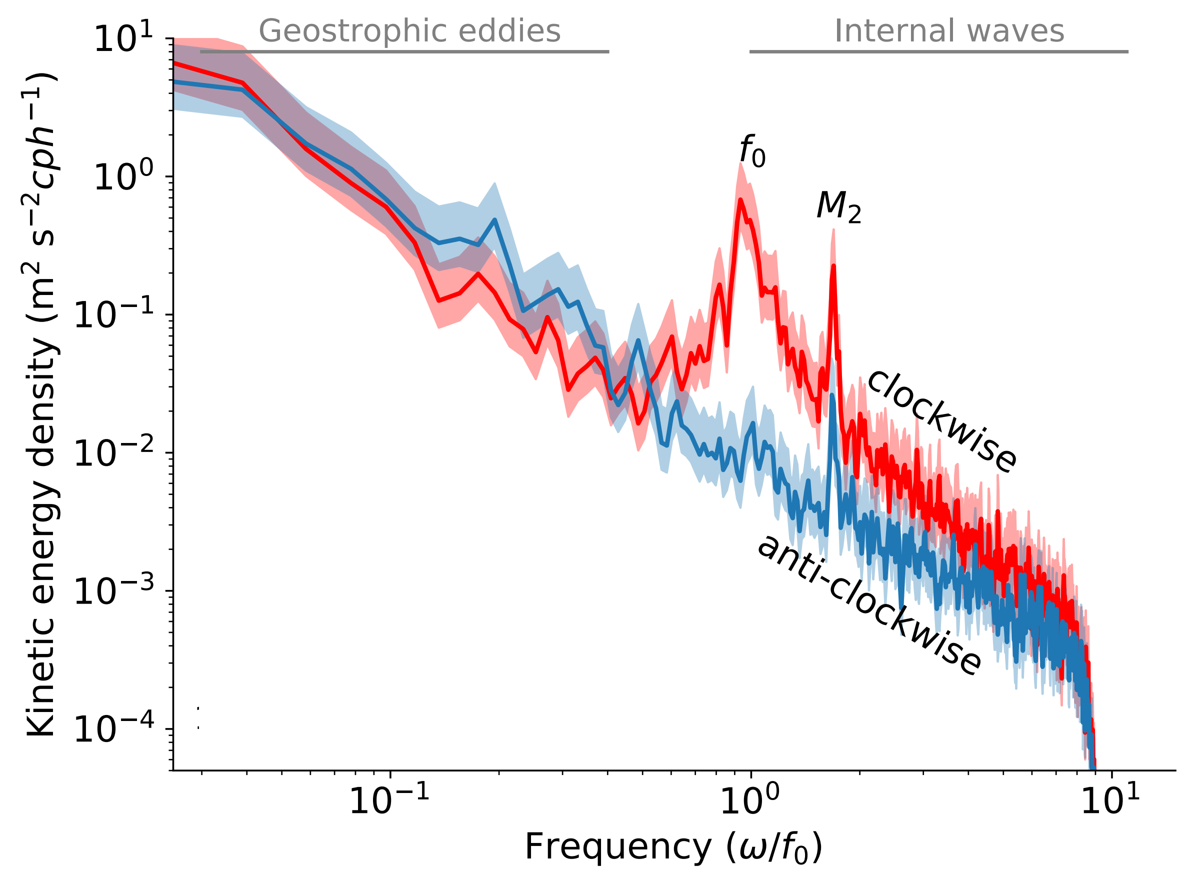
$$A_t + J(\Psi, A) + \tfrac{\mathrm{i}}{2}\triangle\Psi A = \tfrac{\mathrm{i}}{2}\,\underbrace{\frac{f}{m'^2}}_{\hbar}\,\triangle A$$

Negative vortices are analogous to potential wells. NIWs are trapped in negative vortices (and expelled from positive vortices).



In regions of negative vorticity the internal wave band is wider.

$$\omega_{\min} = f + \tfrac{1}{2}\triangle\Psi < f$$



Action conservation

An obvious conservation law:

$$\mathsf{L}A^* \left[\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \tfrac{\mathrm{i}}{2}\triangle\Psi\, \mathsf{L}A + \tfrac{\mathrm{i}}{2} f \triangle A = 0\right]$$
$$\Rightarrow \partial_t |\mathsf{L}A|^2 + \boldsymbol{\nabla}\boldsymbol{\cdot}\boldsymbol{F} = 0\,.$$

So NIW horizontal kinetic energy is conserved?

$$|\mathsf{L}A|^2 = u^2 + v^2$$

Yes — this is consistent with Bretherton&Garrett!

$$\text{Action} = \frac{\text{energy}}{\text{intrinsic frequency}} \quad \overset{\text{NIW}}{\longmapsto} \quad \frac{u^2 + v^2}{2f}$$

$$\triangle \stackrel{\text{def}}{=} \partial_x^2 + \partial_y^2\,, \qquad \mathsf{L} \stackrel{\text{def}}{=} \partial_z \tfrac{f^2}{N^2}\partial_z$$

This concludes derivation and discussion of the NIW equation

$$\partial_{t_2} \mathsf{L}A + J(\Psi, \mathsf{L}A) + \tfrac{\mathrm{i}}{2} \triangle \Psi \, \mathsf{L}A + \tfrac{\mathrm{i}}{2} f \triangle A = 0$$

Now let’s solve the NIW equation and explain how and why there can be significant vertical propagation of NIW energy into the deep ocean.

$$\triangle \stackrel{\text{def}}{=} \partial_x^2 + \partial_y^2 \,, \qquad \mathsf{L} \stackrel{\text{def}}{=} \partial_z \frac{f^2}{N^2} \partial_z$$

Gill's problem

A storm impulsively forces a shallow layer of ocean.

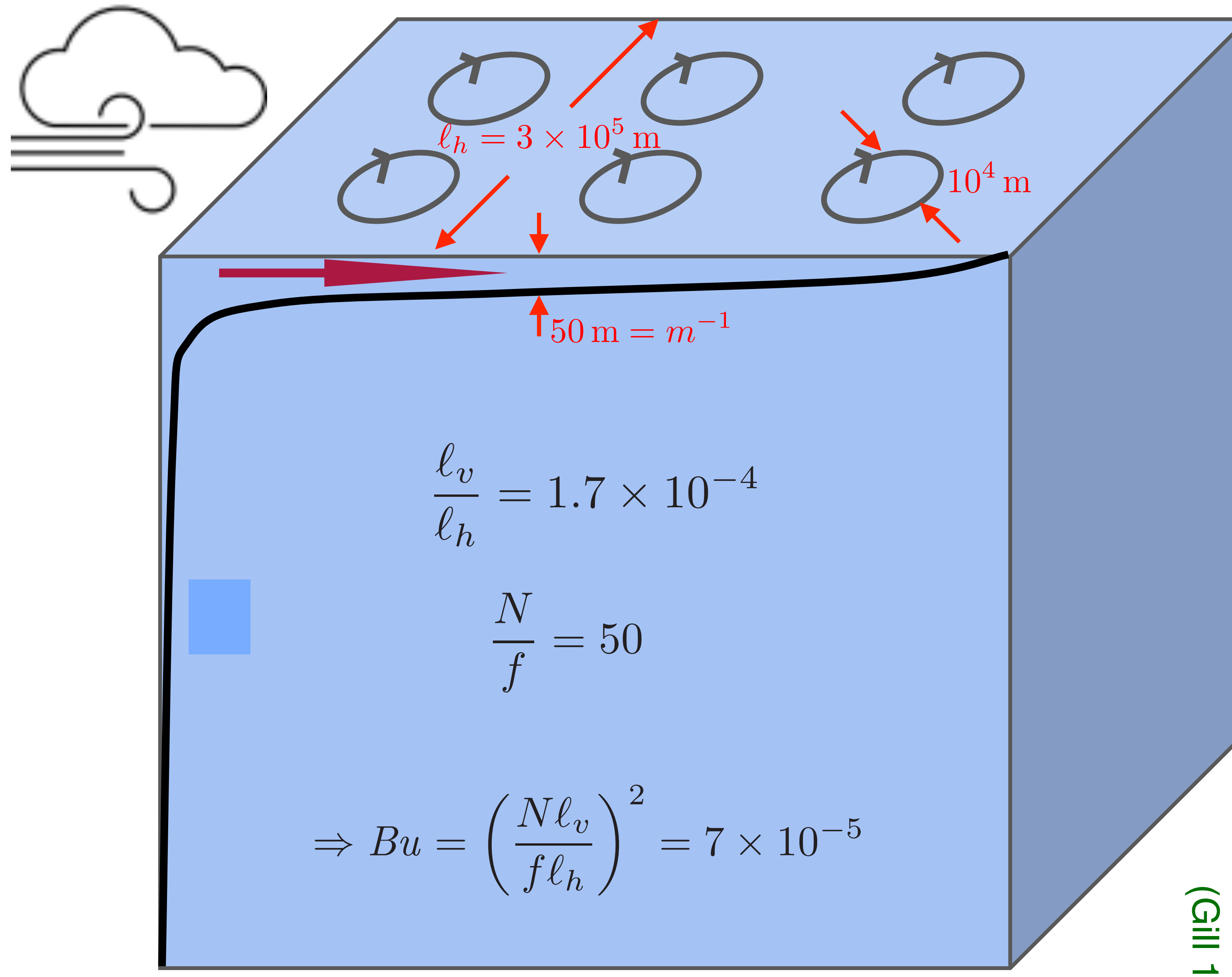
But the NIW vertical group velocity is:

$$\omega \approx f + \frac{N^2k^2}{2fm^2}$$

$$\Rightarrow \qquad c_\text{v} \approx -\frac{N^2k^2}{fm^3} \approx -Bu\frac{f}{m}$$

The time for vertical transit through the surface layer is:

$$\frac{\ell_v}{c_\text{v}} = \frac{1}{f\,Bu},$$
$$= 15000/f$$
$$= 4.5\ \text{years}$$



(Gill 1984)

The aspect ratio is too extreme, and therefore the radiation damping time scale is too long by a factor ~ 1000

Four resolutions of Gill's problem

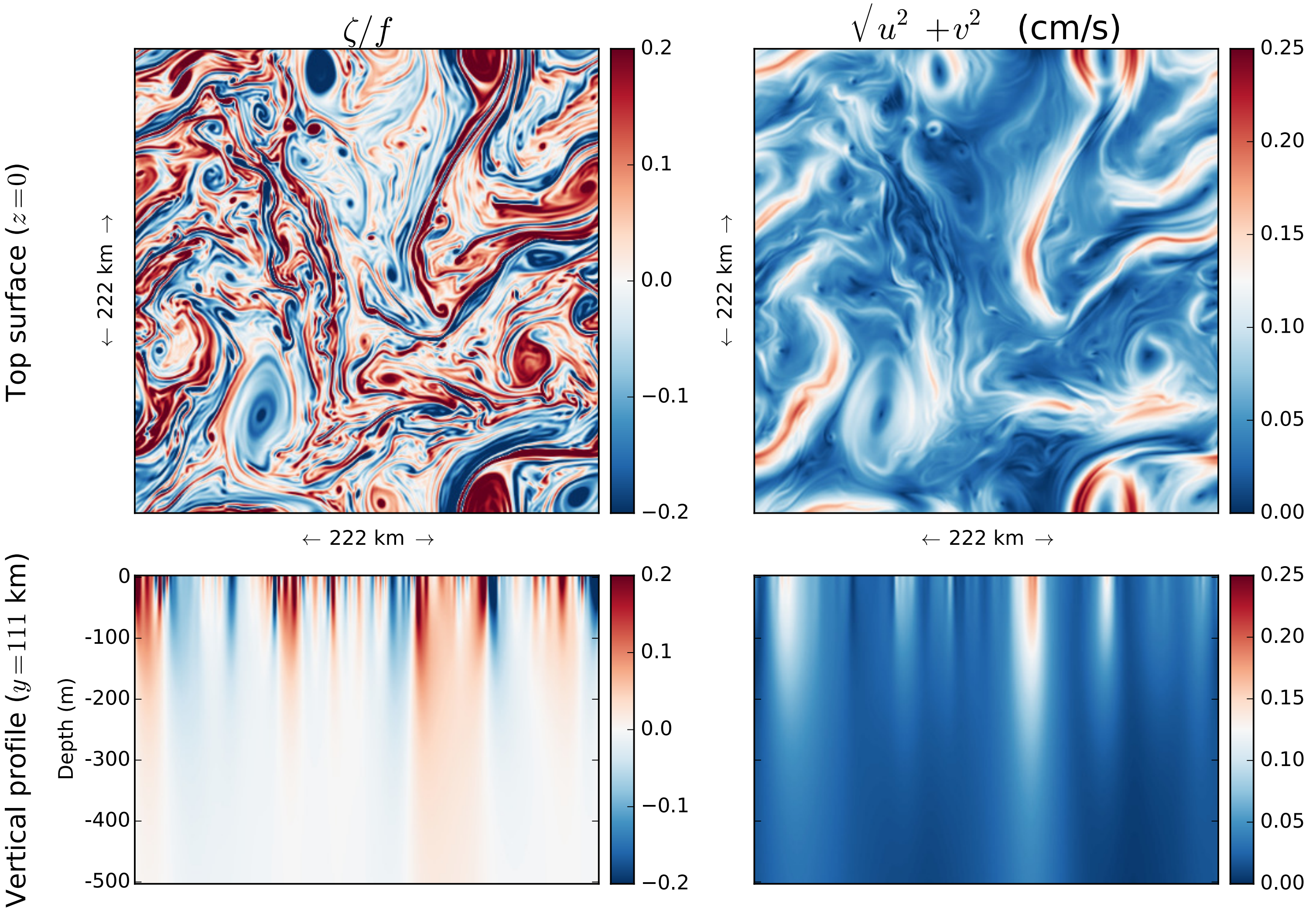
NIWs might have small (10-100km) horizontal length scales because:

(a) The atmospheric forcing has small (~100km) scales.

(b) The mixed-layer depth is not uniform in the horizontal e.g., fronts.

(c) The $\boldsymbol{\beta}$-effect reduces the initial 1000km scale.

(d) Mesoscale eddies "imprint" their length scales on the NIWs

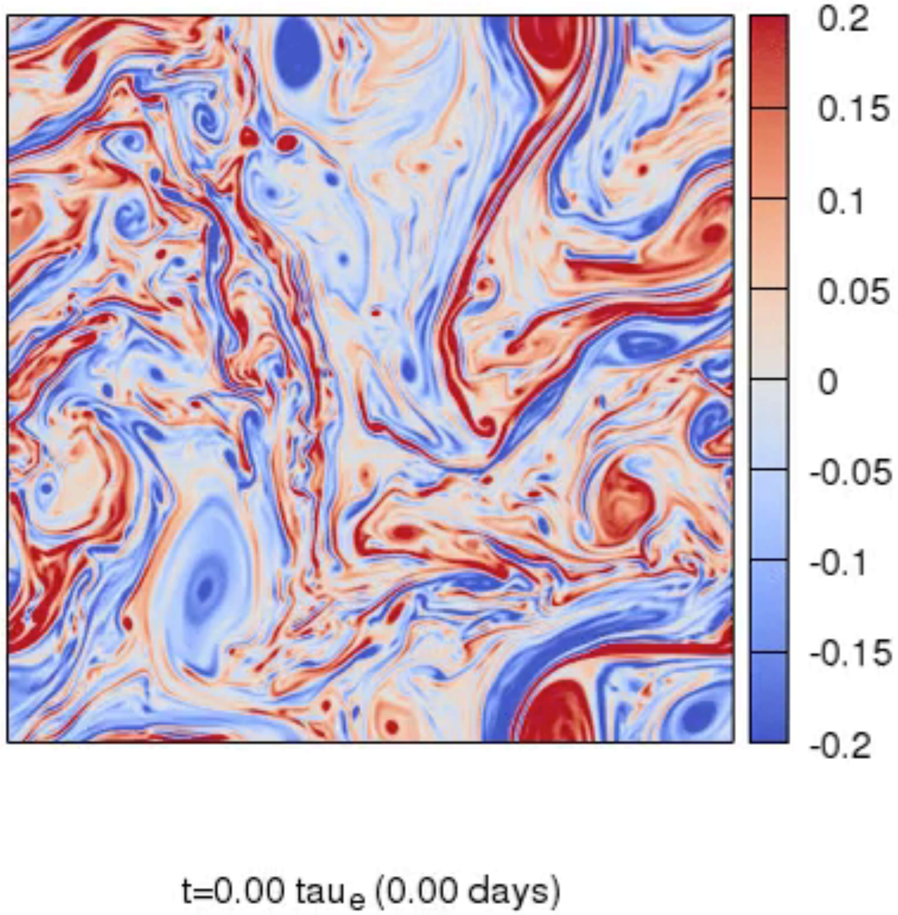


$$\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \underbrace{\tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A}_{\text{"imprinting"}} + \tfrac{\mathrm{i}}{2} f \triangle A = 0$$

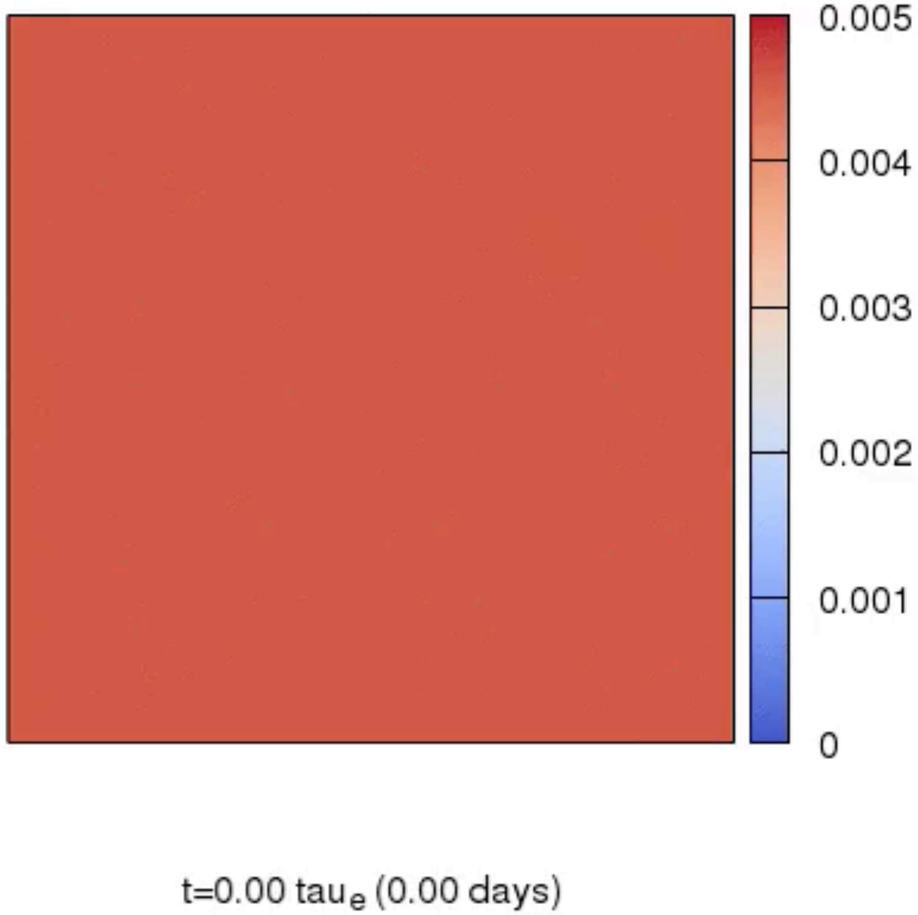
A numerical solution

$$\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \underbrace{\tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A}_{\text{“imprinting”}} + \tfrac{\mathrm{i}}{2} f \triangle A = 0$$

sea-surface $\triangle\Psi$



sea-surface $u^2 + v^2 = |\mathsf{L}A|^2$



t=0: mature geostrophic turbulence

$$t = 0: \quad u + \mathrm{i}v = \mathsf{L}A = \tilde{U}\exp\left(-\frac{z^2}{\delta^2}\right)$$

$$Q_t + J(\psi, Q) = 0$$

$$Q = \Psi_{xx} + \Psi_{yy} + \left(\frac{f^2}{N^2}\Psi_z\right)_z$$

By 15 days there is no NIW energy left at the sea-surface!

A numerical solution

$$\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \underbrace{\tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A}_{\text{“imprinting”}} + \tfrac{\mathrm{i}}{2} f \triangle A = 0$$

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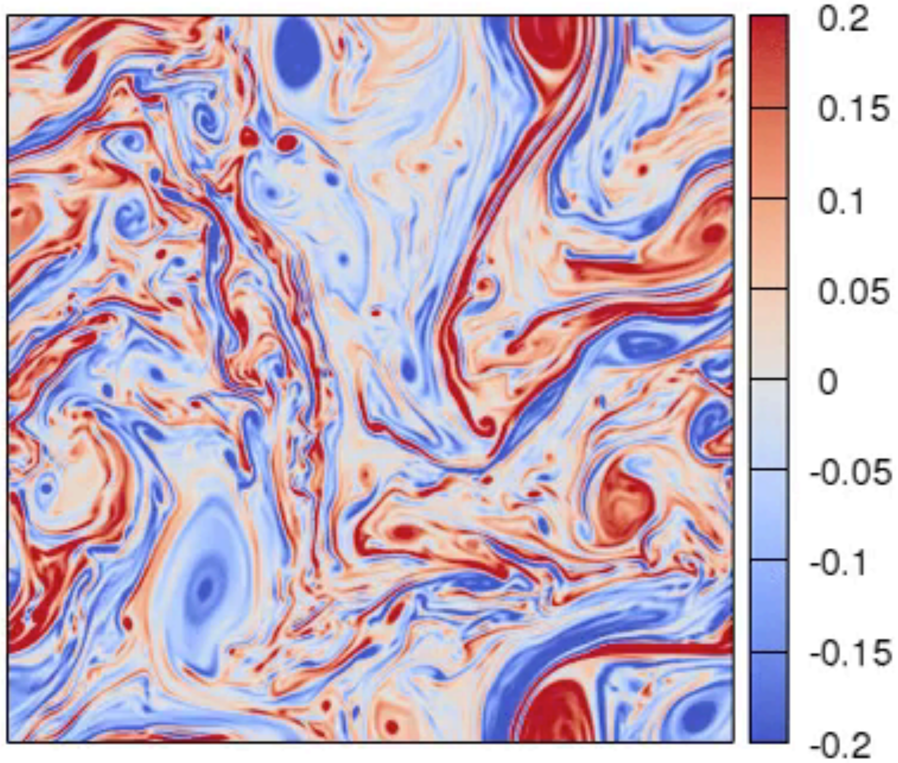

t=0.00 tau$_e$ (0.00 days)

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$$Q_t + J(\psi, Q) = 0$$

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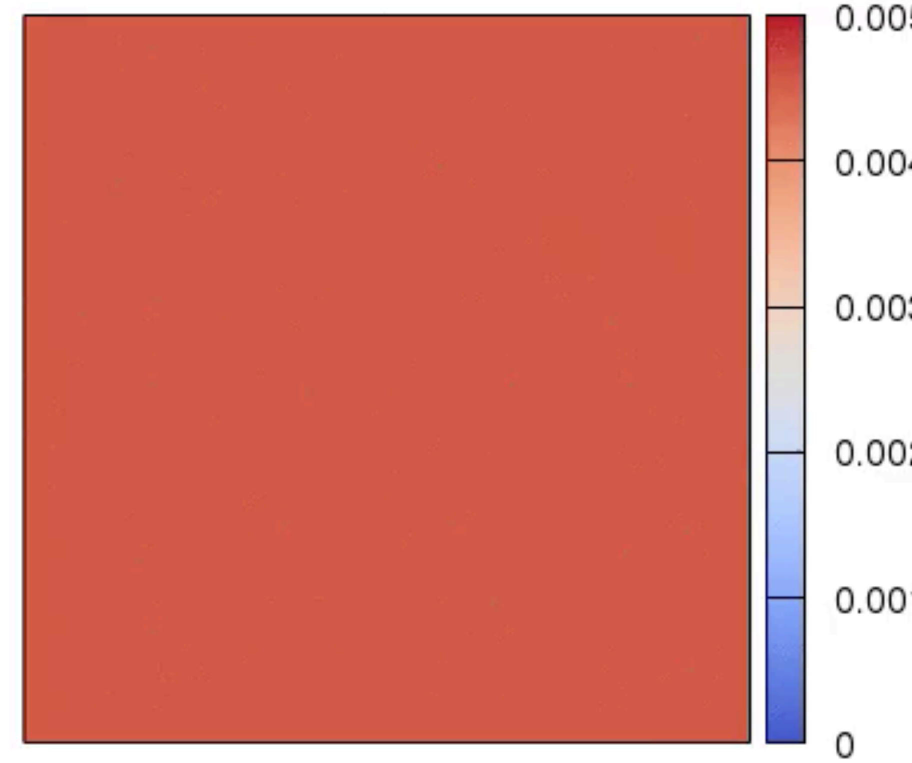

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By 15 days there is no NIW energy left at the sea-surface!

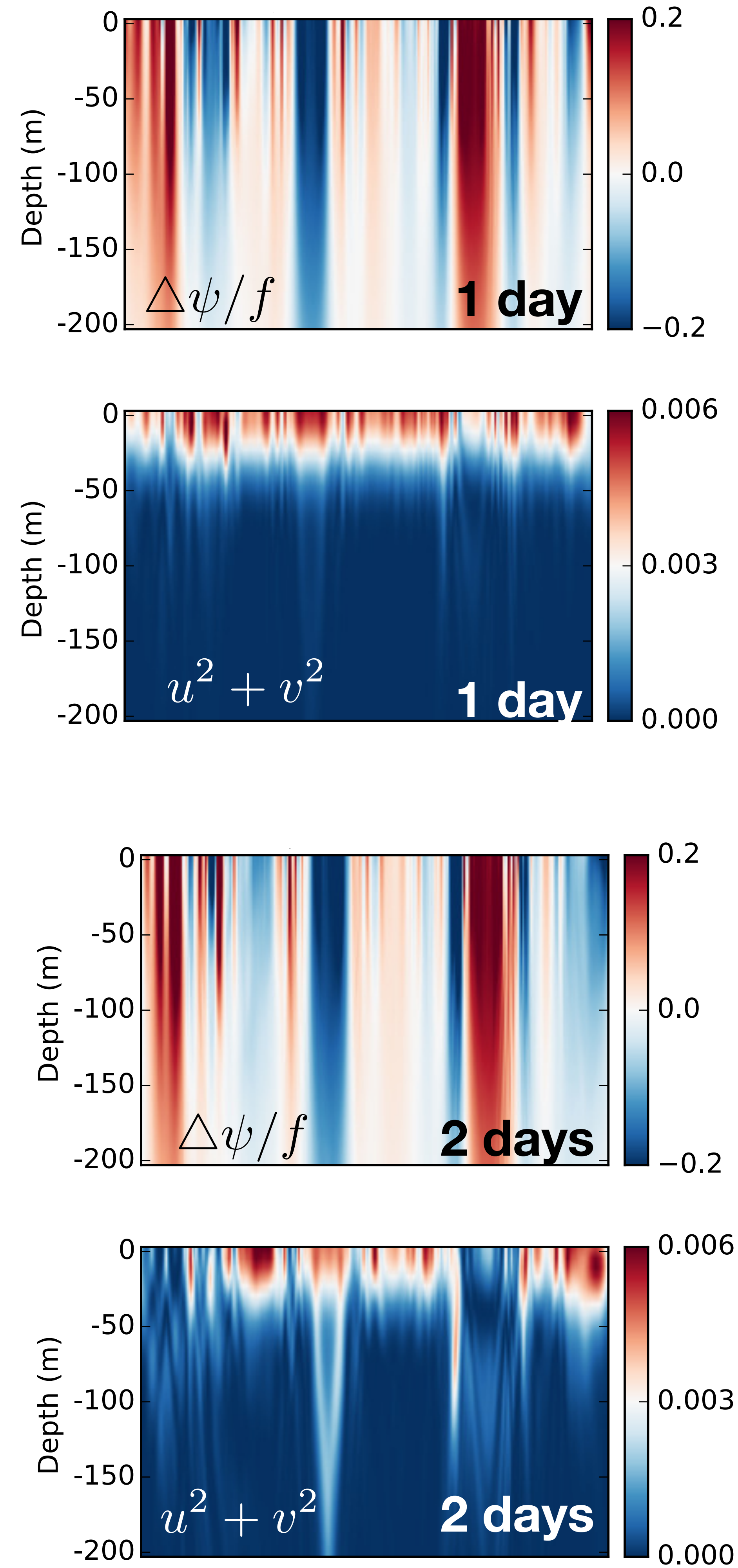
What happened to the WKE?

WKE is rapidly expelled from positive vortices and concentrated into negative vortices.

Then there is rapid downwards vertical propagation inside the negative vortices.

Negative vortices are NIW wave guides, or inertial drain pipes. Negative vortices are vertical conduits to the deep ocean.

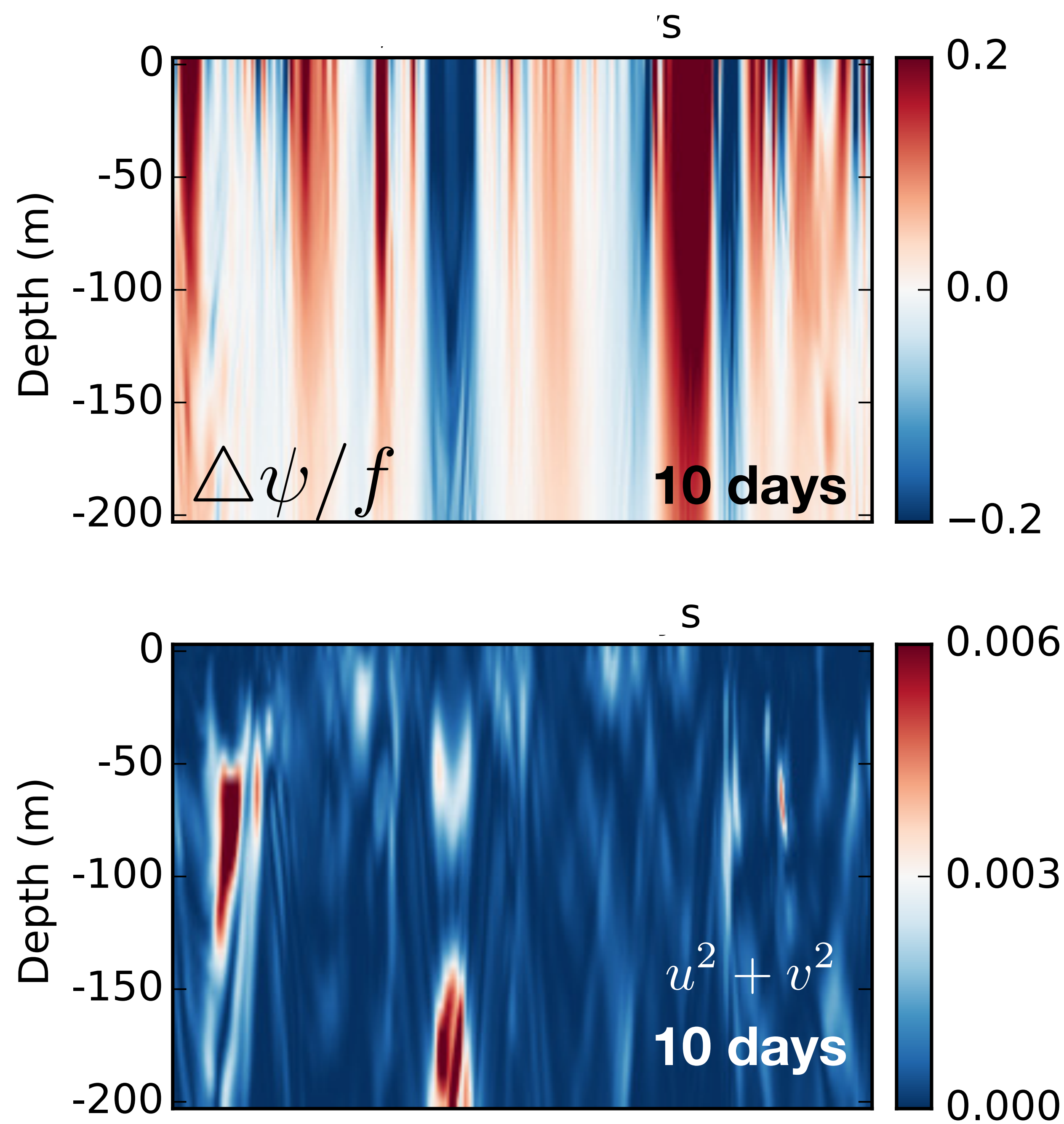
Lee & Niiler (1998)



Lee & Niiler (1998)

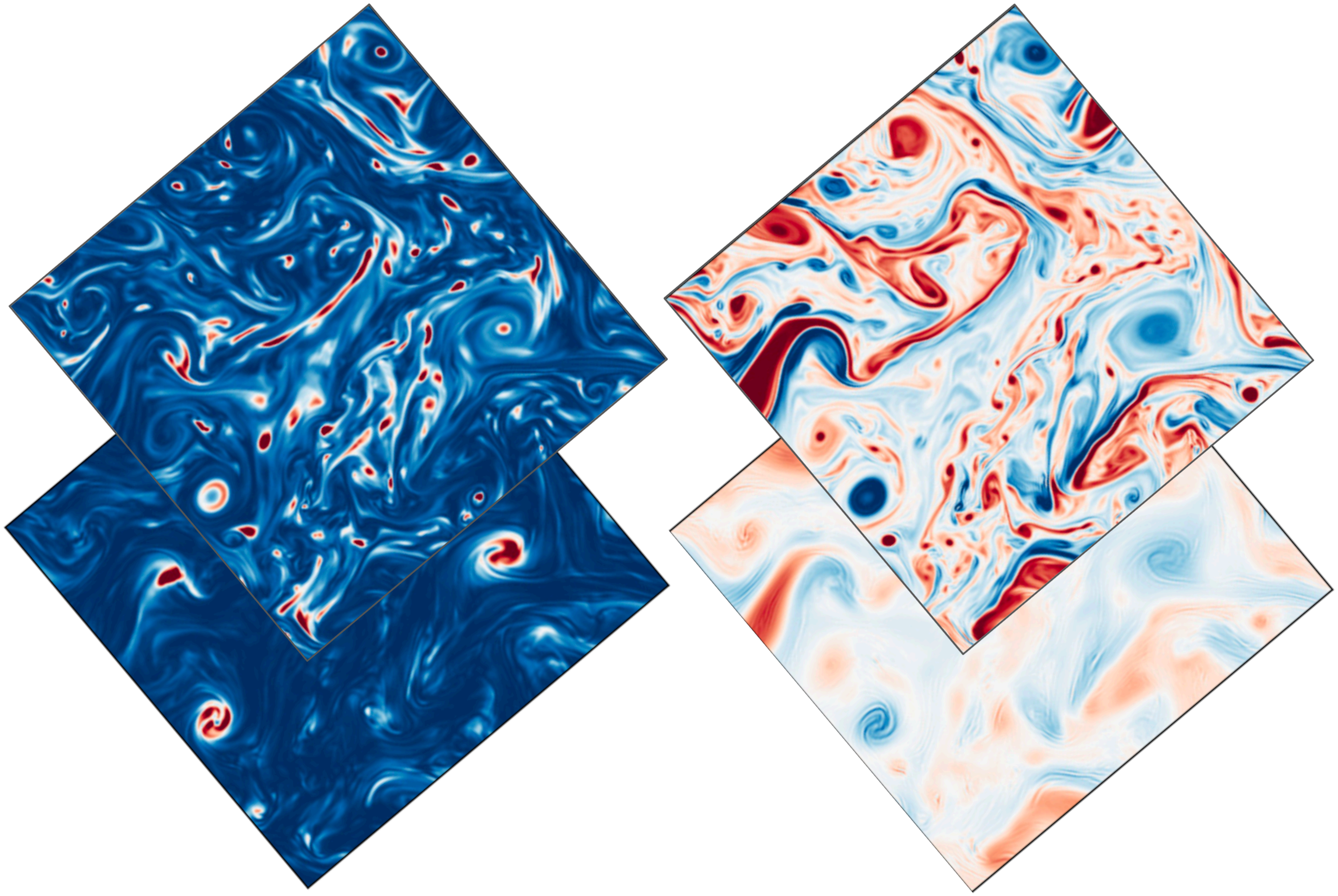
“Inertial drain pipes”

Negative vortices are vertical conduits that drain NIWs into the deep ocean.



(I’m re-branding Lee & Niiler’s “inertial chimneys” into “inertial drain pipes”.)

This concludes the discussion of Gill's problem



What have I not talked about?

Do NIWs affect eddies?
Is there energy transfer?

So far we have considered the NIWs as a dynamically passive (complex) scalar.

$$\partial_t \mathsf{L}A + J(\Psi, \mathsf{L}A) + \tfrac{\mathrm{i}}{2}\triangle\Psi\,\mathsf{L}A + \tfrac{\mathrm{i}}{2}f\triangle A = 0$$

The QG approximation is used for the eddy field.

$$Q_t + J(\Psi, Q) = 0$$
$$Q = \triangle\Psi + \mathsf{L}\Psi$$

Xie & Vanneste (2015) have included the reaction of the NIWs back onto the QG eddies.

Recall the back-rotated velocity

$$u + \mathrm{i}v = \mathrm{e}^{-\mathrm{i}ft}\mathsf{L}\,A$$

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~~$Q = \triangle\Psi + \mathsf{L}\Psi$~~

Xie & Vanneste (2015) have included the reaction of the NIWs back onto the QG eddies.

$$Q = \triangle\Psi + \mathsf{L}\Psi + \tfrac{1}{4f}\triangle|\mathsf{L}A|^2 + \tfrac{\mathrm{i}}{2f} J(\mathsf{L}A^*, \mathsf{L}A)$$

Recall the back-rotated velocity

$$u + \mathrm{i}v = \mathrm{e}^{-\mathrm{i}ft}\mathsf{L}\,A$$

A special case: The vertical plane wave model

The vertical plane wave simplification:
(very similar to the shallow-water model)

L $\mapsto \left(\frac{fm}{N}\right)^2$

$$u + \mathrm{i}v = \mathrm{e}^{\mathrm{i}(mz - ft)}\phi - \psi_y + \mathrm{i}\psi_x$$

The NIW equation is now:
(2D quantum analogy again)

$$\phi_t + J(\psi, \phi) + \frac{\mathrm{i}\triangle\psi}{2}\phi = \frac{\mathrm{i}\eta}{2}\triangle\phi$$

PV conservation

$$q_t + J(\psi, q) = 0$$

The PV is:

$$q = \triangle\psi + \frac{1}{4f}\triangle|\phi|^2 + \frac{\mathrm{i}}{2f}J(\phi^*, \phi)$$

This is the simplest system exhibiting the interaction between geostrophic turbulence and NIWs.

Why can be be done with this model, other than numerical solution?

$$q_t + J(\psi, q) = 0$$

$$\phi_t + J(\psi, \phi) + \frac{\mathrm{i}\triangle\psi}{2}\phi = \frac{\mathrm{i}\eta}{2}\triangle\phi$$

$$q = \triangle\psi + \frac{1}{4f}\triangle|\phi|^2 + \frac{\mathrm{i}}{2f}J(\phi^*, \phi)$$

Two conservation laws

Wave action, equivalently NIW kinetic energy is still conserved.

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \tfrac{1}{2}|\phi|^2\rangle = 0$$

NIW KE

But "coupled energy" is also conserved.

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \tfrac{1}{2}|\boldsymbol{\nabla}\psi|^2 + \tfrac{1}{4}\lambda^2|\boldsymbol{\nabla}\phi|^2\rangle = 0$$

Eddy KE NIW PE

Xie & Vanneste (2015)

$$u + \mathrm{i}v = \mathrm{e}^{\mathrm{i}(mz-ft)}\phi - \psi_y + \mathrm{i}\psi_x \qquad \Rightarrow \qquad u^2 + v^2 = |\boldsymbol{\nabla}\psi|^2 + |\phi|^2$$

Stimulated Loss of Balance
also known as SLoB

$$\underbrace{\left\langle \tfrac{1}{2}|\boldsymbol{\nabla}\psi|^2 \right\rangle}_{\text{eddy KE}} + \underbrace{\left\langle \tfrac{1}{2}\lambda^2|\boldsymbol{\nabla}\phi|^2 \right\rangle}_{\text{NIW PE}} = \text{constant}$$

If NIW PE increases, then eddy KE must decrease.
This is *stimulated* transfer of eddy KE to NIWs.
(There is also *spontaneous* transfer.)

Xie & Vanneste (2015)

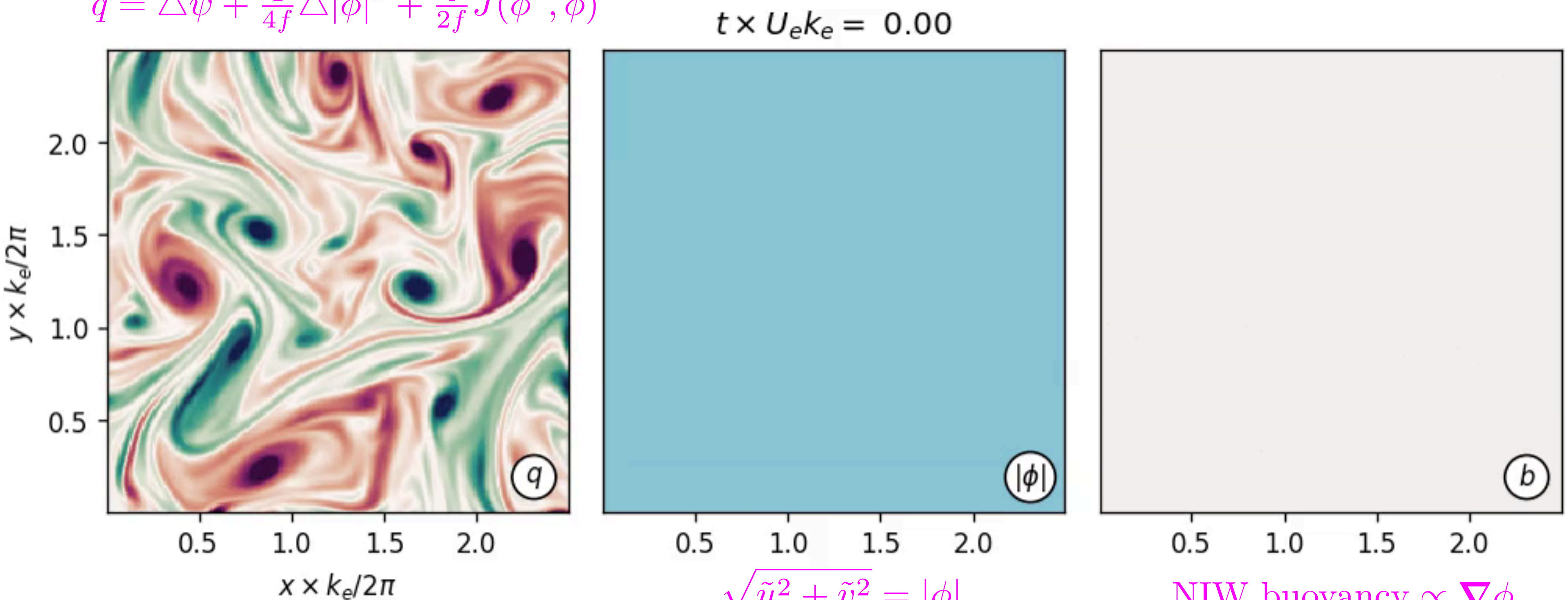
And here are two good reasons for NIW PE to increase.

$$\phi_t + J(\psi, \phi) + \frac{\mathrm{i}\triangle\psi}{2}\phi = \frac{\mathrm{i}\eta}{2}\triangle\phi$$

1. advection 2. refraction

A solution of the single-wave model

$q = \triangle\psi + \frac{1}{4f}\triangle|\phi|^2 + \frac{\mathrm{i}}{2f}J(\phi^*, \phi)$



$\sqrt{\tilde{u}^2 + \tilde{v}^2} = |\phi|$

NIW buoyancy $\propto \boldsymbol{\nabla}\phi$

t=0: (u,v)= mature 2D turbulence + a spatially uniform NIW

$\boldsymbol{\phi}$ develops small-scale structure: SLoB works.

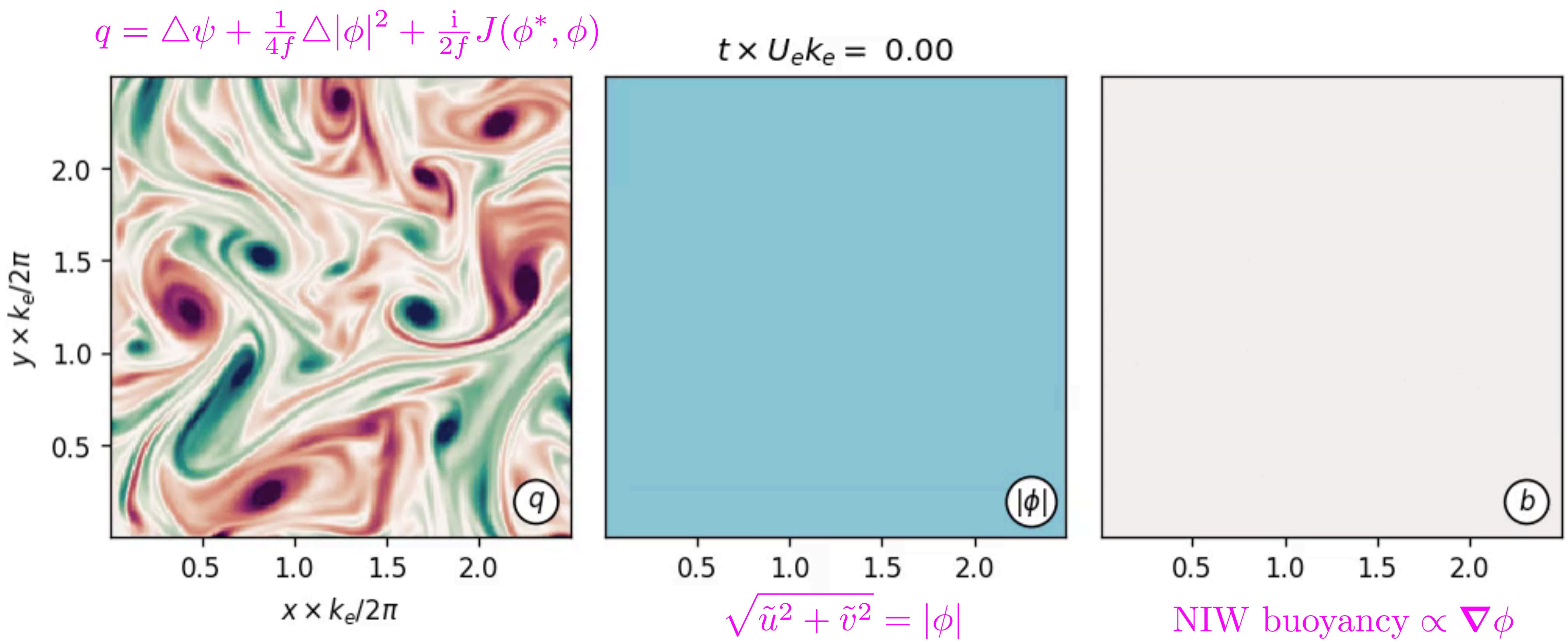
But SLoB is much slower than e^t

In other words, $\boldsymbol{\phi}$ does not cascade to high wavenumber.

→ Conclusion: SLoB is slow.

• Rocha, Wagner & Young (JFM 2018)

A solution of the single-wave model



t=0: (u,v)= mature 2D turbulence + a spatially uniform NIW

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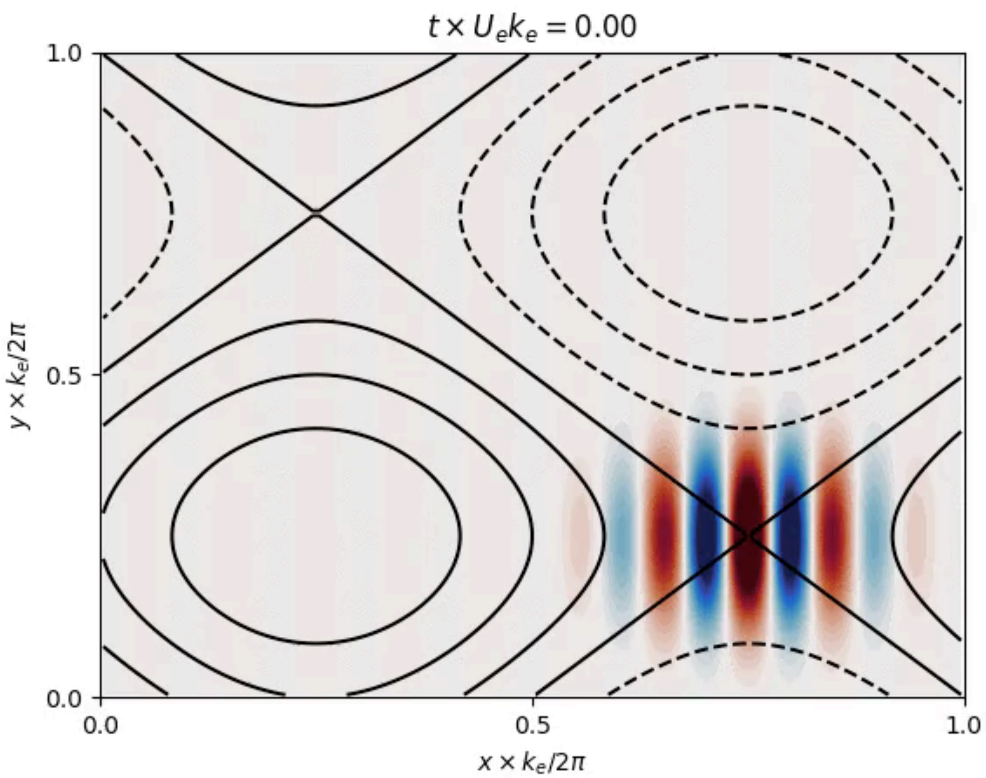
➡ Conclusion: SLoB is slow.

• Rocha, Wagner & Young (JFM 2018)

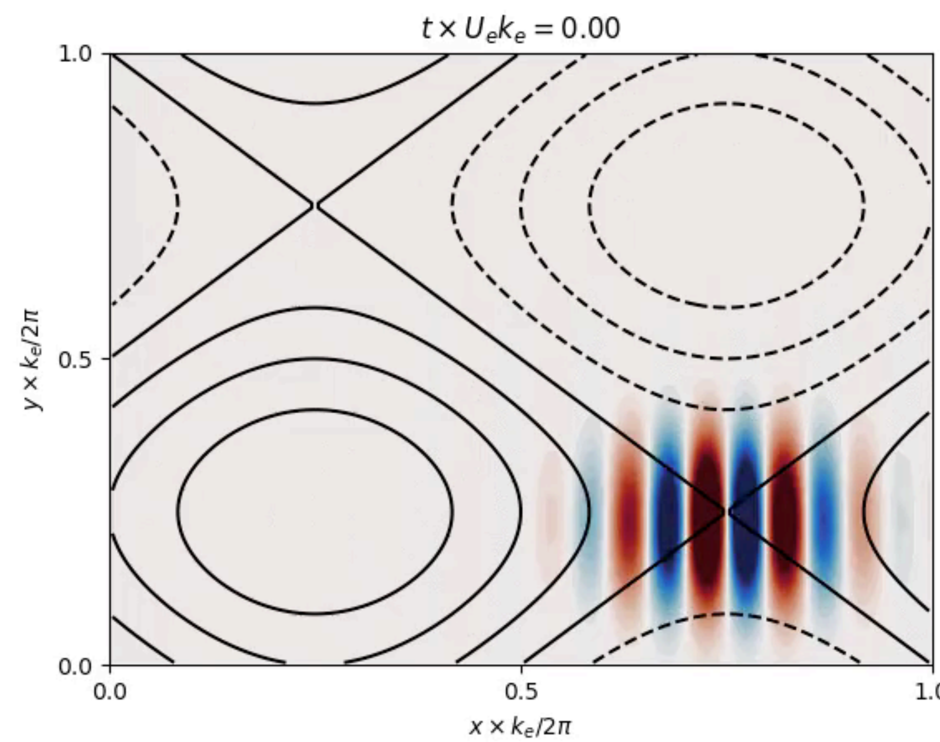
Q: Why is SLoB slow?
A: Wave escape.

Rocha, Wagner & Young (JFM 2018)

Passive Scalar



NIWs



$$\theta_t + J(\psi, \theta) = \kappa \triangle \theta$$

$$\phi_t + J(\psi, \phi) + \frac{\mathrm{i}\triangle\psi}{2}\phi = \frac{\mathrm{i}\eta}{2}\triangle\phi = 0$$

If $\boldsymbol{\nabla\Phi}$ increases, then so does the group velocity and thus waves accelerate out of straining regions.

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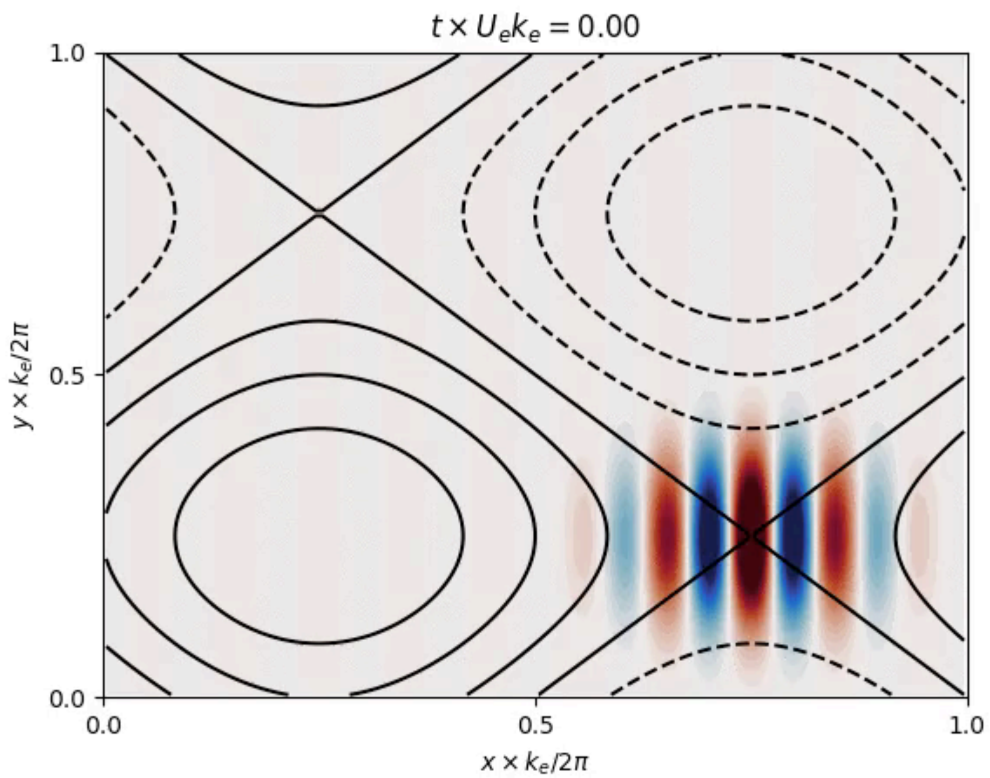
Rocha, Wagner & Young (JFM 2018)

Passive Scalar



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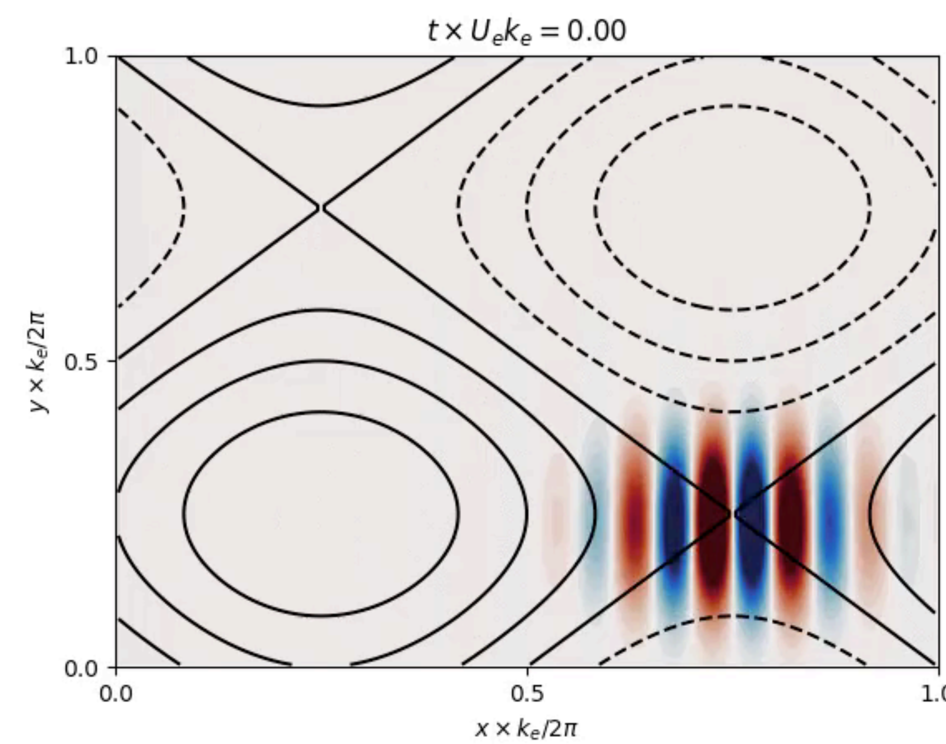
NIWs



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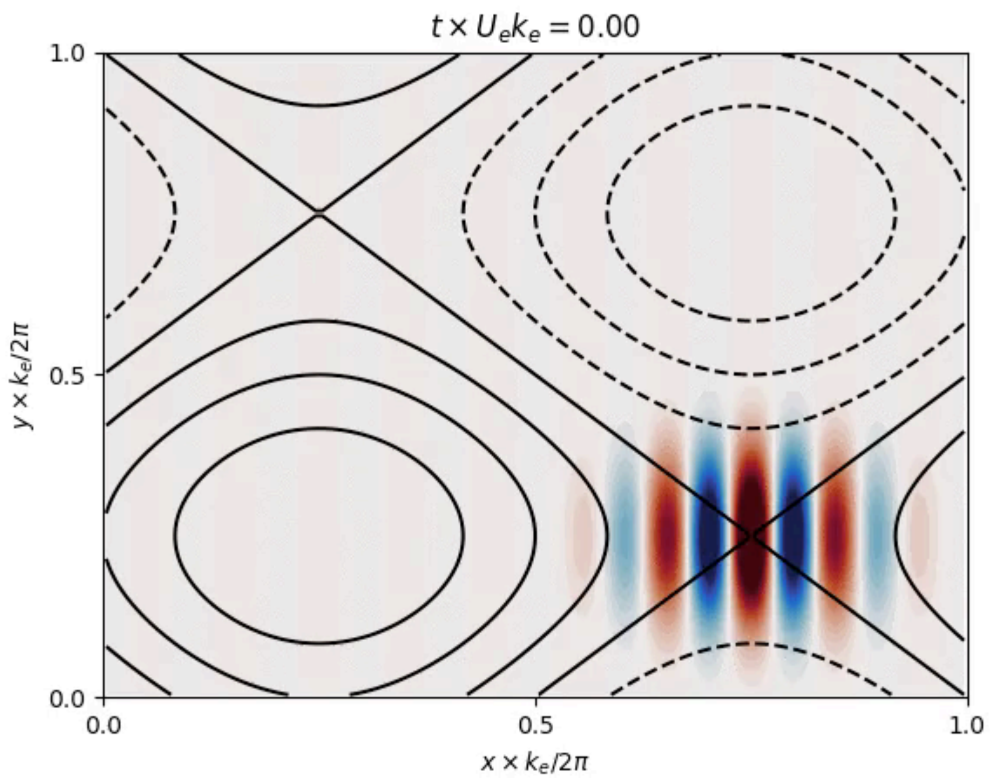

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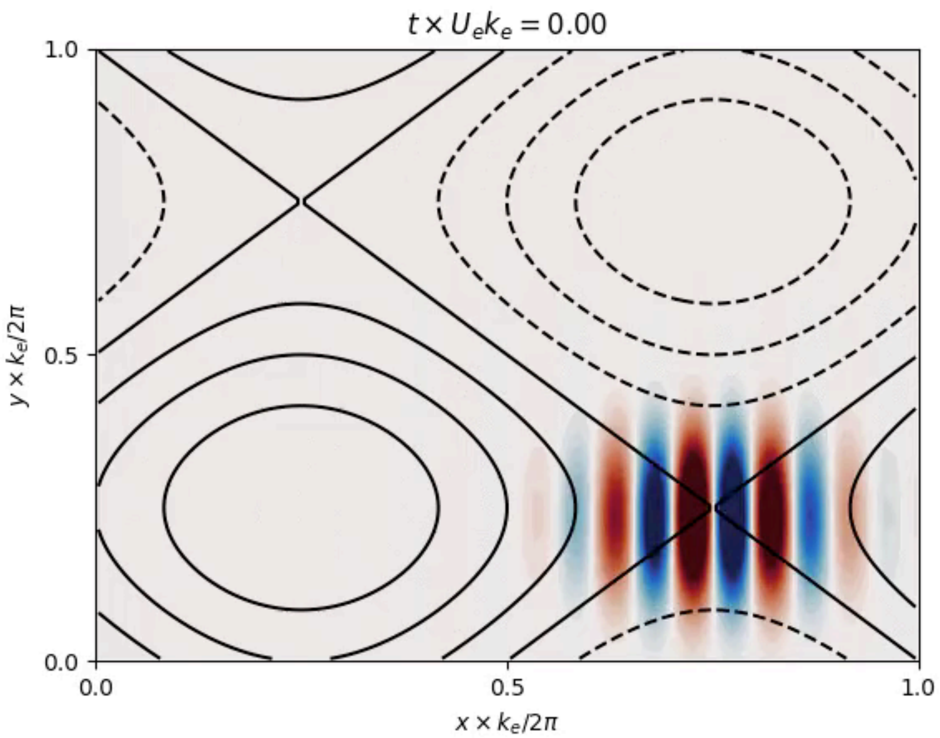
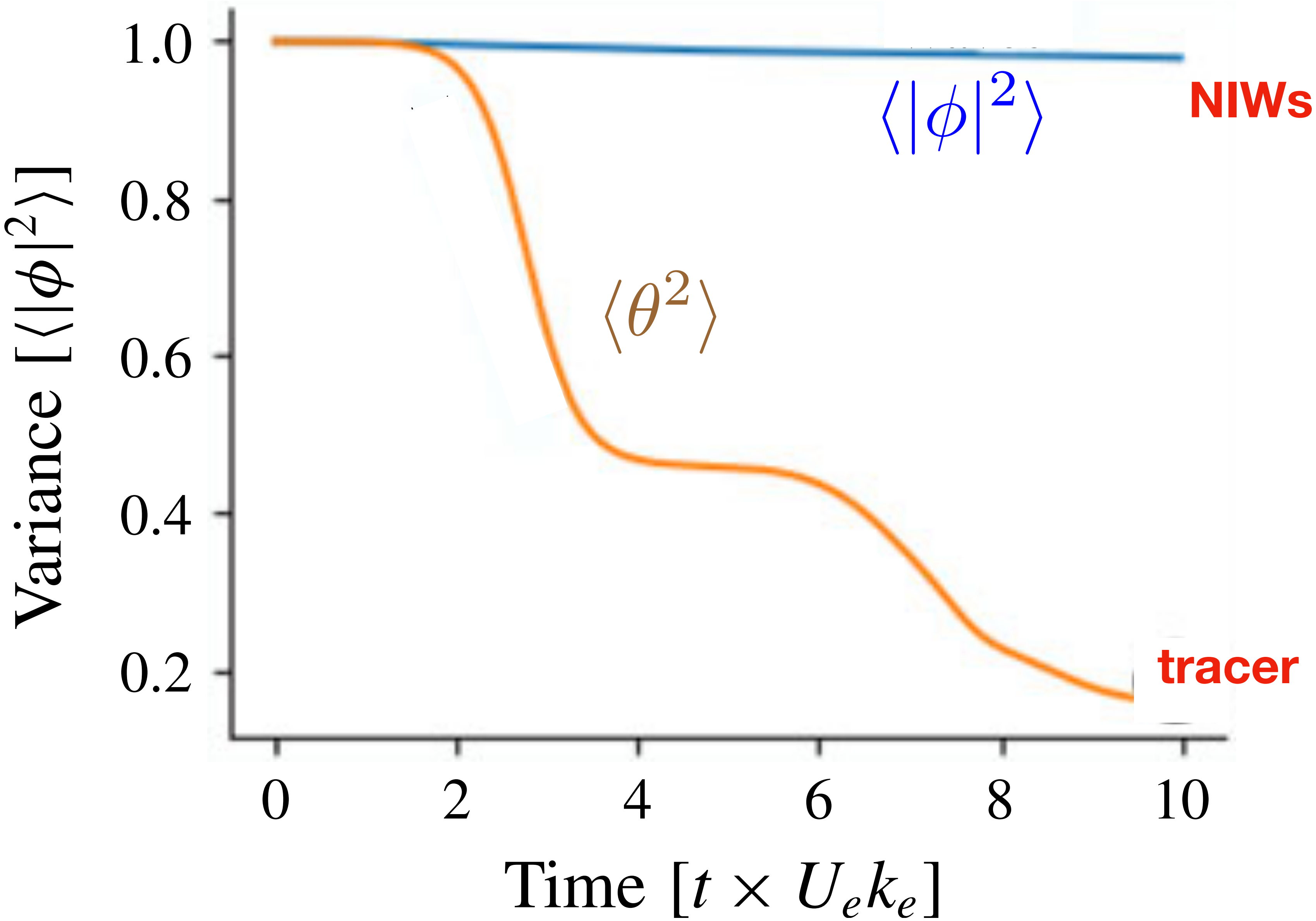
NIWs



$$\phi_t + J(\psi, \phi) + \frac{\mathrm{i}\triangle\psi}{2}\phi = \frac{\mathrm{i}\eta}{2}\triangle\phi = 0$$

If $\nabla\Phi$ increases, then so does the group velocity and thus waves accelerate out of straining regions.

$\theta_t + J(\psi, \theta) = \kappa \triangle \theta$ **versus** $\phi_t + J(\psi, \phi) + \frac{\mathrm{i} \triangle \psi}{2} \phi = \frac{\mathrm{i}\eta}{2} \triangle \phi$

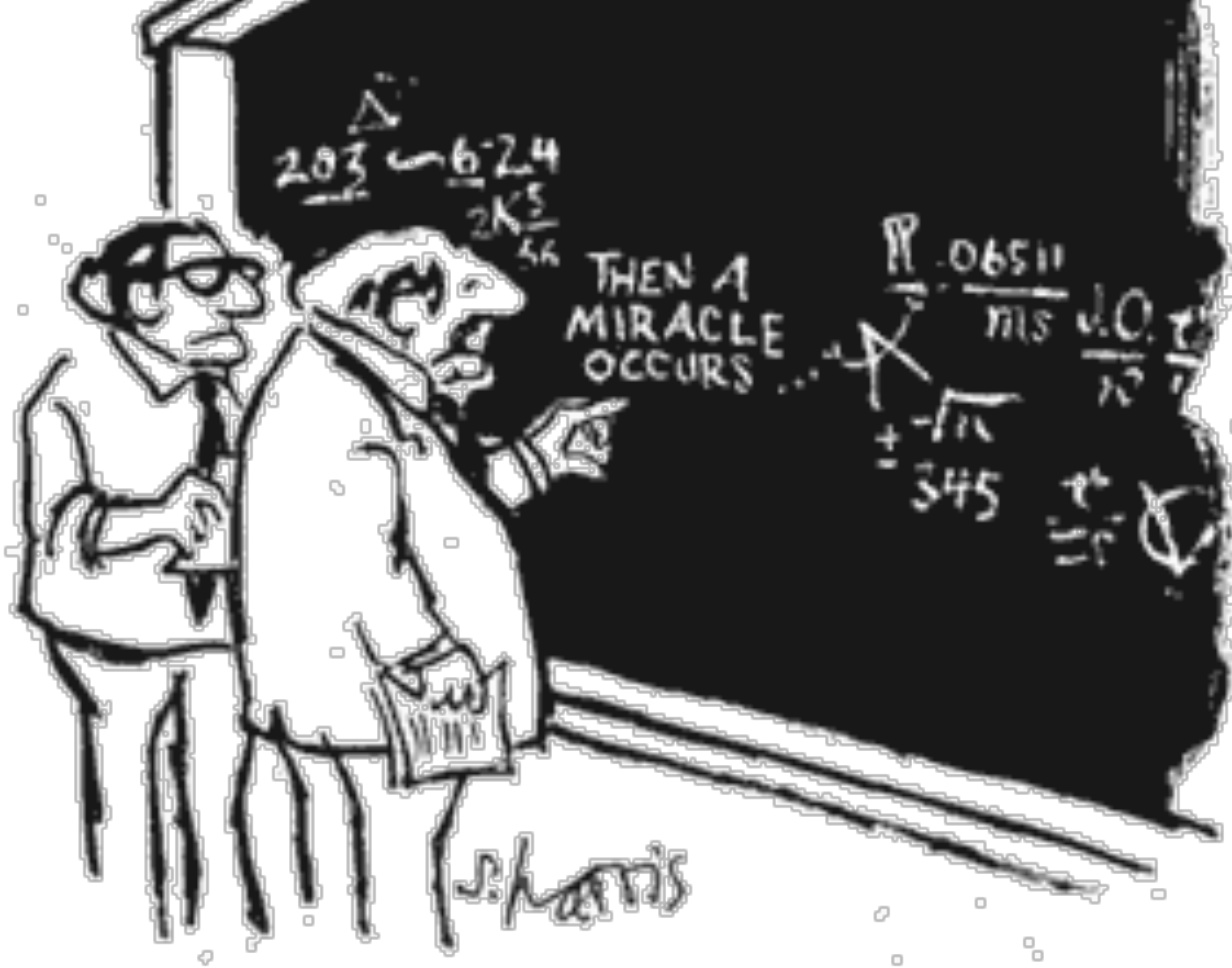


$\text{NIW KE} = \frac{1}{2}\langle|\phi|^2\rangle$ $\text{NIW PE} = \frac{1}{4}\lambda^2\langle|\nabla\phi|^2\rangle$

SLoB is slow.
$\nabla\Phi$ does not increase exponentially with time.

NIWs resist straining by the eddy velocity.
This is wave escape powered by dispersion.

The energetics of SLoB is confusing e.g., Reynolds stresses produce NIW PE.



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Sydney Harris

THE END