

SIO203B/MAE294B Mid-term 2025

You can bring a one-page “cheat sheet” with notes.
All problems have equal weight.

Problem 1

(i)) Find two terms in the $\epsilon \rightarrow 0$ expansion of the three roots of

$$\epsilon x^3 + x - 1 = 0. \quad (1)$$

(ii) With $\eta = 1/\epsilon$, consider the other limit $\epsilon \rightarrow \infty$. Find two terms in the $\eta \rightarrow 0$ expansion of the three roots.

Problem 2

Find the leading-order $x \rightarrow \infty$ behaviour of

$$E_{1/3}(x) = \int_x^\infty \frac{e^{-t}}{t^{1/3}} dt. \quad (2)$$

You should justify neglect of the remainder in the limit $x \rightarrow \infty$.

Problem 3

Consider

$$\epsilon u_{xx} - (4 + x)u = -(1 - x), \quad (3)$$

posed on the interval $0 < x < 1$ with boundary conditions $u(0) = 0$ and $u(1) = 0$. Find the leading-order $\epsilon \rightarrow 0$ solution in both the interior and the boundary layers and construct a uniform solution.

Turn the page for problem 4

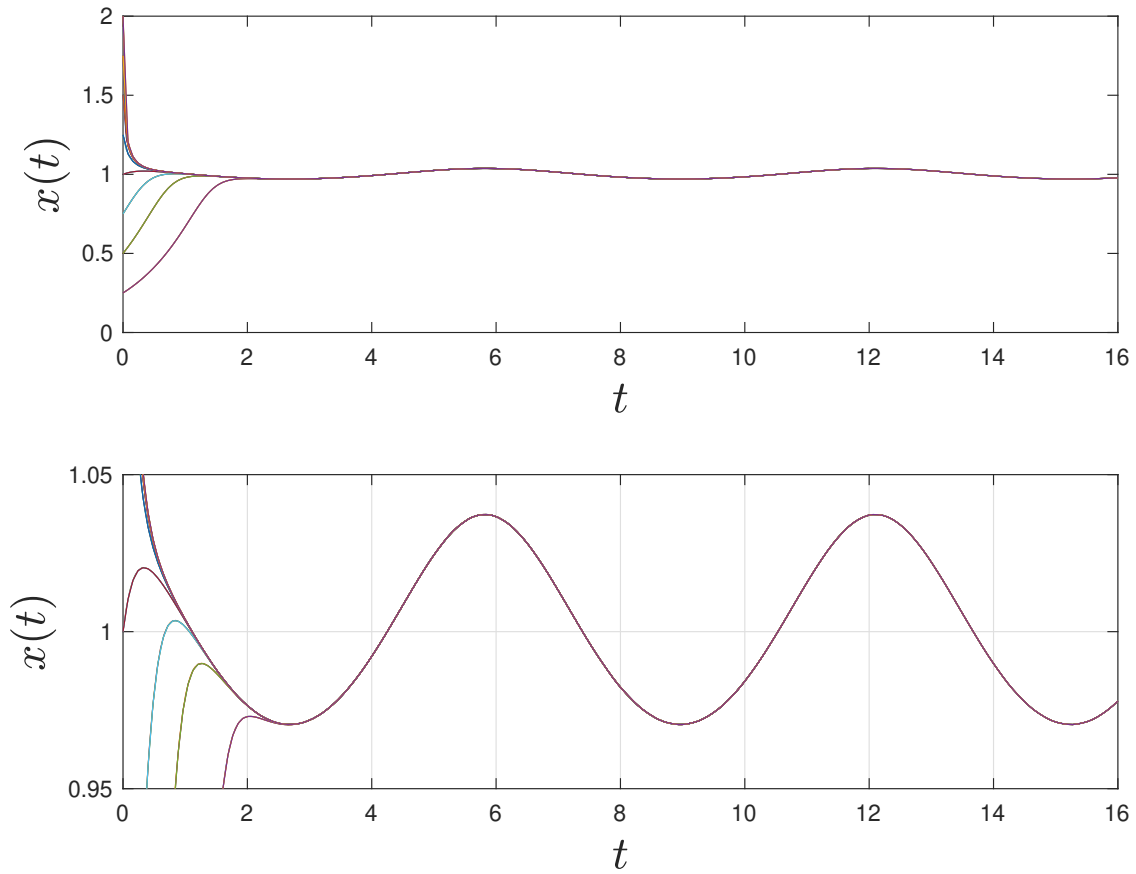


Figure 1: Evolution of 7 initial conditions. The bottom panel is an expanded view showing small oscillations about $x = 1$ at long time.

Problem 4

The figure shows a solution of one of the following differential equations

$$\dot{P} = -P + (1 - \epsilon \cos t)P^7, \quad \dot{Q} = -Q + (1 - \epsilon \cos 2t)Q^7, \quad (4)$$

$$\dot{M} = M - (1 - \epsilon \cos t)M^7, \quad \dot{N} = -N + (1 - \epsilon \cos 2t)N^7. \quad (5)$$

Above $\epsilon \ll 1$. (i) Which differential equation is solved in figure? (Lucky guesses don't count – explain your answer in ten or twenty words.) (ii) Estimate the value of ϵ used to make the figure.